On the Localization Properties of Quantum Field Theories with Infinite Spin

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1 Introduction

2 Compact Localization

3 No-Go Theorem

4 Limit of Representations

5 Summary & Outlook

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- Infinite Spin Representations
- Modular Localization
- String-Localized Fields

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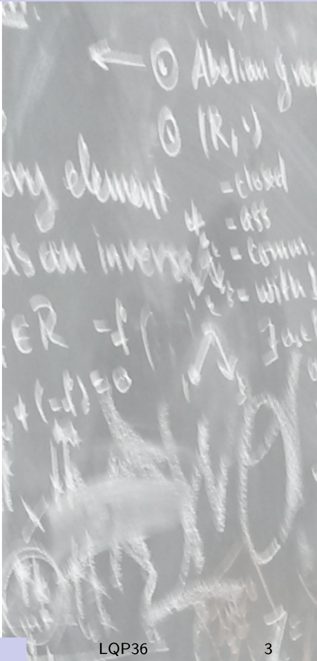

Minkowski space & Poincaré group

- Minkowski space $\mathbb{M} := (\mathbb{R}^4, \eta)$, $\eta = \mathrm{diag}(1, -1, -1, -1)$
- lightcone coordinates $x_\pm := x^0 \pm x^3$, $\mathrm{x} := x^1 + \mathrm{i}x^2$
- matrix form of $x, p \in \mathbb{M}$ ($\sigma_0 := \mathbf{1}$, σ_i: Pauli matrices)

$$\underset{\sim}{x} := \begin{pmatrix} x_+ & \overline{\mathrm{x}} \\ \mathrm{x} & x_- \end{pmatrix} = \sigma_\mu x^\mu,\ \widetilde{p} := \begin{pmatrix} p_- & -\overline{\mathrm{p}} \\ -\mathrm{p} & p_+ \end{pmatrix} \Rightarrow px = \frac{1}{2}\mathrm{Tr}\,\widetilde{p}\underset{\sim}{x}$$

- Poincaré group (unit component) $\mathcal{P}_+^\uparrow = \mathrm{SO}(1,3) \ltimes \mathbb{M}$
- covering group $\mathcal{P}^c = \mathrm{SU}(2) \ltimes \mathbb{M} \xrightarrow{\Lambda} \mathcal{P}_+^\uparrow$

$$(\Lambda(A)x)_{\sim} := A\underset{\sim}{x}A^\dagger,\ (p\Lambda(A))^{\sim} = A^\dagger\widetilde{p}A$$

- irreducible representations on one-particle Hilbert space $\mathcal{H}_1 \rightarrow$

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Irreducible representations: Translation operators

- $U(a) = \mathrm{e}^{\mathrm{i}Pa}$, momentum operator P
- representation property $U(A)U(a)U(A)^\dagger = U(\Lambda(A)a)$

$$\Rightarrow U(A)PU(A)^\dagger = p\Lambda(A) \Rightarrow \mathrm{sp}\, P \text{ is Lorentz-invariant}$$

- Casimir operator $P^2 = m^2\mathbf{1}$ (Schur's Lemma)
- positive energy representations: ($P^0 > 0$)
  - $m > 0$: upper mass-shell

    $H_m^+ = \{p \in \mathbb{M} : \ p^2 = m^2, p^0 > 0\}$
  - $m = 0$: boundary of the forward light cone

    $\partial V^+ = \{p \in \mathbb{M} : \ p_+p_- = \mathrm{p}^2, p^0 > 0\}$

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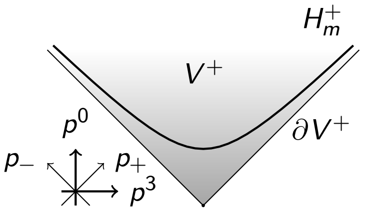

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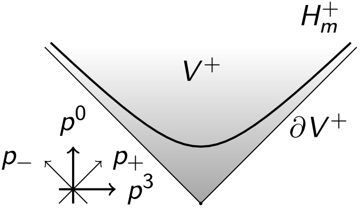

Irreducible representations: Little group construction

- choose reference momentum $q \in \operatorname{sp} P$
- little group $G_q := \operatorname{stab} q = \{R \in \mathrm{SL}(2,\mathbb{C}) : \; q\Lambda(R) = q\}$
- representation D on Hilbert space $\mathcal{H}_q$
- $m > 0$: massive representations
  - $q_m := (m, \vec{0}) \in H_m^+$ (rest frame)
  - $\operatorname{stab} q_m = \mathrm{SU}(2)$
  - D: spin s representation, $\mathcal{H}_{q_m} = \mathbb{C}^{2s+1}$
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  - $q_0 := (\frac{1}{2}, \frac{1}{2}\vec{e}_3) \in \partial V^+$
  - $\operatorname{stab} q_0 = \widetilde{E(2)} \xrightarrow{\lambda} E(2)$ (covering of 2d Euclidean group)

$$\widetilde{E(2)} = \{[\varphi, a] \in \mathrm{SL}(2,\mathbb{C}) : \; \varphi \in \mathbb{R}, a \in \mathbb{R}^2\}$$

$$[\varphi, a] = \begin{pmatrix} e^{i\varphi} & \\ a & e^{-i\varphi} \end{pmatrix}$$

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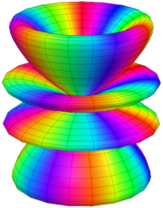

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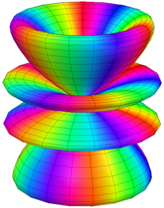

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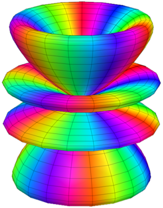

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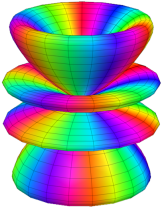

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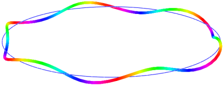

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Irreducible representations: One-particle space

- Wigner boost B_p with $q\Lambda(B_p) = p$:

$$B_p := \begin{cases} \sqrt{\frac{\widetilde{p}}{m}} & m > 0 \\ \frac{1}{\sqrt{p_-}} \begin{pmatrix} p_- & \overline{\mathrm{p}} \\ & 1 \end{pmatrix} & m = 0 \end{cases}$$

- Wigner rotation $R(A, p) = B_p A B^{-1}_{p\Lambda(A)} \in \operatorname{stab} q$

representation of $\mathrm{SL}(2, \mathbb{C})$ on $\mathcal{H}_1 := L^2(\operatorname{sp} P) \otimes \mathcal{H}_q$

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Tomita operator for wedges

- Standard wedge $W_0 := \{x \in \mathbb{M} : \pm x_\pm > 0\}$
- $\Delta^{it} := U_1(\mathrm{e}^{-\pi\sigma_3 t})$ subgroup of boosts preserving W_0
- reflection $(R_{W_0}x)_\pm = -x_\pm$, $J := U(R_{W_0})$ complex conjugation
- Tomita operator $S_{W_0} := J\Delta^{\frac{1}{2}}$ (domain restricted by required analytic continuation)

real subspace for the standard wedge

$$\mathcal{K}_1(W_0) := \{\psi \in \mathrm{dom}\Delta^{\frac{1}{2}} : S_{W_0}\psi = \psi\}$$

- extension to arbitrary wedges by covariance:

$$\mathcal{K}_1(W) := U_1(A,a)\mathcal{K}_1(W_0) \text{ for } W = \Lambda(A)W_0 + x$$

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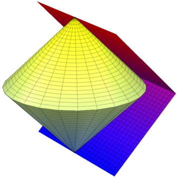
Real subspaces for arbitrary regions

- subspace for $\mathcal{O} \subset \mathbb{M}$

$$\mathcal{K}(\mathcal{O}) := \bigcap_{W \supset \mathcal{O} \text{ wedge}} \mathcal{K}(W)$$



real subspace $\mathcal{K} \subset \mathcal{H}_1$ is standard iff

- $\mathcal{K} \cap \mathrm{i}\mathcal{K} = 0$ (separating)
- $\overline{\mathcal{K} + \mathrm{i}\mathcal{K}} = \mathcal{H}$ (cyclic)

- $\tilde{\mathcal{O}} \subset \mathcal{O}' := \{\tilde{x} \in \mathbb{M} : (\tilde{x} - x)^2 < 0 \; \forall x \in \mathcal{O}\}$

$$\Rightarrow \mathcal{K}(\tilde{\mathcal{O}}) \perp \mathcal{K}(\mathcal{O}) \text{ wrt. } \Im \circ \langle \cdot, \cdot \rangle$$

- $\mathcal{K}(C)$ is standard for $C \subset \mathbb{M}$ a spacelike cone [Brunetti, Guido, Longo '02]

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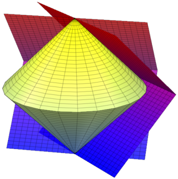

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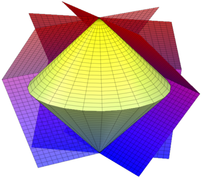

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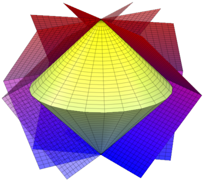

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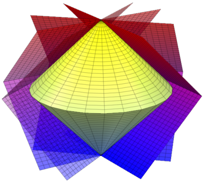

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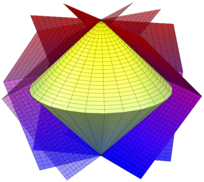

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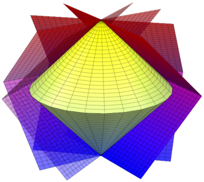

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Definition

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is called an *intertwiner*, if

- $D(R(A,p))u(p\Lambda(A), e) = u(p, \Lambda(A)e)$ (intertwiner eq)
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1. pullback representation on G_q-orbits [Mund, Schroer, Yngvason '06]
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String-localized one-particle states

- conjugate intertwiner: $u_c(p,h) := Ju(-pR_{W_0}, (R_{W_0})_* h)$
- u has distributional boundary value in e.
- Single particle vectors $\psi_{(c)}(f,h) \in \mathcal{H}_1$ are defined by

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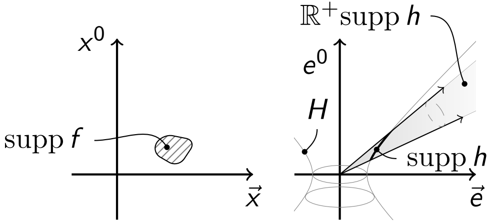

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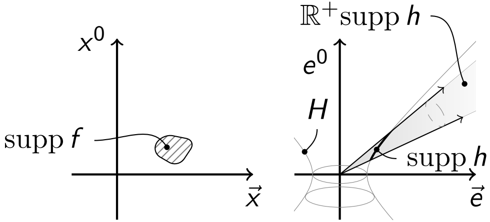

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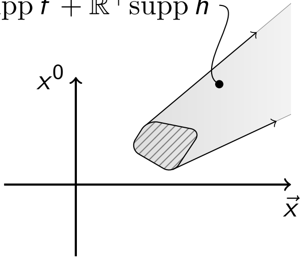

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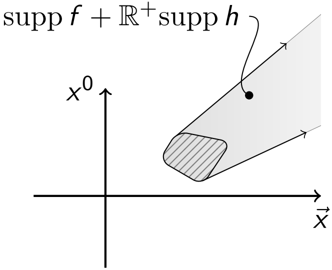

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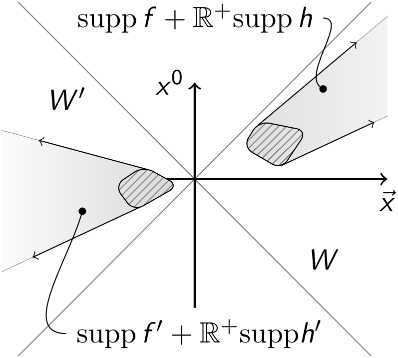

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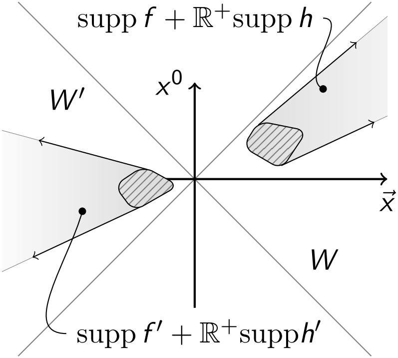

1 Introduction

2 Compact Localization
- Two-Particle States
- Candidates for Two-Particle Observables

3 No-Go Theorem

4 Limit of Representations

5 Summary & Outlook

- Dependency on semi-infinite string-direction is intrinsic for infinite spin-case → No-Go Thm.
  [Yngvason '70] [Longo, Morinelli, Rehren '15]

Construction of two-particle intertwiners [MSY '06]

Let $F \in \mathcal{S}(\mathbb{R})$ and define $u_2 : (\partial V^+)^{\times 2} \to \mathcal{H}_q^{\otimes 2}$ by

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Localized two-particle wavefunctions (cf. MSY '06,)

Let $\mathcal{O} \subset \mathbb{M}$ compact and $g \in \mathcal{S}(\mathbb{M}^{\times 2})$ real-valued with $\operatorname{supp} g \subset \mathcal{O}^{\times 2}$. If $u_2 \in L^2_{\mathrm{loc}} \otimes \mathcal{H}^{\otimes 2}$ is polynomially bounded, i.e.

$$||u_2(p, \tilde{p})||_{\mathcal{H}_q^{\otimes 2}} \leq M(p, \tilde{p})$$

with M a polynomial, then the function

$$\psi(p, k, \tilde{p}, \tilde{k}) := \tilde{g}(p, \tilde{p}) u_2(p, \tilde{p})(k, \tilde{k})$$

is modular localized in $\mathcal{O}$, which means

$$\psi \in \mathcal{K}_2(\mathcal{O})$$

with the two-particle subspace $\mathcal{K}_2$ defined via second quantization of the operators S_W.

Proposed construction of two-particle observables [MSY '06]

Candidate observables are of the form

$$B(g) := \int \widetilde{\mathrm{d}p} \int \mathrm{d}\nu(k) \int \widetilde{\mathrm{d}\tilde{p}} \int \mathrm{d}\nu(\tilde{k})\, \hat{g}(p,\tilde{p}) u_2(p,\tilde{p})(k,\tilde{k})$$
$$a^\dagger(p,k) a^\dagger(\tilde{p},\tilde{k}) + \dots$$

such that $B(g)\Omega \in \mathcal{H}_2$ is a two-particle wavefunction given by

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- Locality in the vacuum expectation value

$$\langle \Omega, [B(g), B(\tilde{g})]\Omega \rangle = 0$$

if $(x - x')^2 < 0\ \forall x \in \mathrm{supp} g, x' \in \mathrm{supp} \tilde{g}$.

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- Relative Commutator
- Restriction of the Integrals
- Analysis of Singularities

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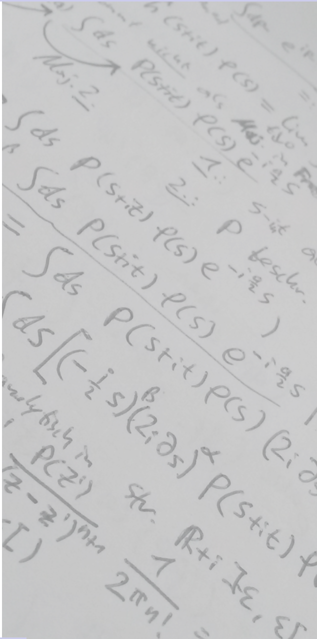

- Q: Existence of nontriv. operators with compact localization?
- →Negative result for the following class of operators on $\mathcal{F}$, motivated by the suggestions in [YMS '06], [Schroer '08].

Definition

An operator-valued distribution B on $\mathcal{S}(\mathbb{M}^{\times 2})$ of the form

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$$\hat{g}(p,\tilde{p}) u_2(p,\tilde{p})(k,\tilde{k}) a^\dagger(p,k) a^\dagger(\tilde{p},\tilde{k})$$
$$+\hat{g}(-p,-\tilde{p}) u_{2c}(p,\tilde{p})(k,\tilde{k}) a(p,k) a(\tilde{p},\tilde{k})$$
$$+\hat{g}(p,-\tilde{p}) u_0(p,\tilde{p})(k,\tilde{k}) a^\dagger(p,k) a(\tilde{p},\tilde{k})$$
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with fixed coefficient functions u_2, u_{2c}, u_0, u_{0c} is called a *Two-particle observable* if... [cf. Streater, Wightman '64, chap. 3]

1. Domain and Continuity
   - For all $g \in \mathcal{S}(\mathbb{M}^{\times 2})$, $B(g)$ is defined on the domain $\mathcal{D}$ of vectors which is spanned by products of the String fields $\Phi(f, h)$ applied to the vacuum Ω.
     By the Reeh-Schlieder Thm., $\mathcal{D}$ is dense in the Fock space $\mathcal{F}$.
   - For fixed vectors $\phi, \psi \in \mathcal{H}$, the assignment

     $$g \in \mathcal{S}(\mathbb{M}^{\times 2}) \mapsto \langle\phi| B(g) |\psi\rangle \in \mathbb{C}$$

     is a tempered distribution, i.e. $g \mapsto B(g)$ is an *operator-valued distribution*.
   - $B(\overline{g}) = B(g)^\dagger$

2. Transformation Law
   - For $p, \tilde{p} \in \partial V^+$ and $A \in \mathrm{SL}(2, \mathbb{C})$, the two-particle intertwiner equation holds almost everywhere in the sense of $\widetilde{\mathrm{d}p}\widetilde{\mathrm{d}\tilde{p}}\mathrm{d}\nu(k)\mathrm{d}\nu(\tilde{k})$:

$$D(R(A, p)) \otimes D(R(A, \tilde{p}))u_2(p\Lambda(A), \tilde{p}\Lambda(A)) = u_2(p, \tilde{p}).$$

   - u_2, u_{2c}, u_0, u_{0c} are locally square-integrable and polynomially bounded.
3. Relative locality
   Let $f \in \mathcal{S}(\mathbb{M})$, $h \in \mathcal{D}(H)$ and $g \in \mathcal{S}(\mathbb{M}^{\times 2})$ such, that

$$(x + \lambda e - y_{1,2})^2 < 0 \;\forall\; x \in \operatorname{supp} f, e \in \operatorname{supp} h, \lambda \in \mathbb{R}^+, \\ (y_1, y_2) \in \operatorname{supp} g.$$

   Then the associated fields commute:

$$[\Phi(f, h), B(g)] = 0$$

One-particle string-intertwiners

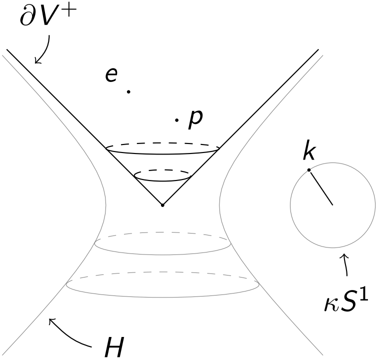
Lemma

Let $u_1(p,e)(k)$ a solution of the one-particle intertwinereq. Then there is a function F_1, defined on the interior of the upper half-plane, such that:

1. The intertwiner u_1 is given by

$$u_1(p,e)(k) = \mathrm{e}^{\mathrm{i}k\cdot\frac{\mathrm{e}-\frac{e_-}{p_-}\mathrm{p}}{2p\cdot e}} F_1(p\cdot e).$$

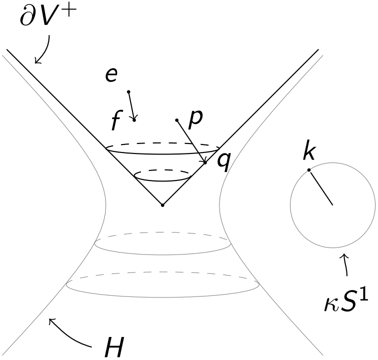
2. A choice of the function F_1 can be made in such a way that u_1 is polynomially bounded in p, analytic in e for $\Im(e) \in V^+$ and bounded by an inverse power at the boundary.



cf. uniqueness proof for string-localized fields [MSY '06, Lemma B 3 ii)]

Step 1

- $A = B_p \in \mathrm{SL}(2, \mathbb{C})$
- $R(B_p^{-1}, p) = \mathbf{1}$
- $u_1(q, \Lambda(B_p)e) = u_1(p, e)$
- $f := \Lambda(B_p)e$



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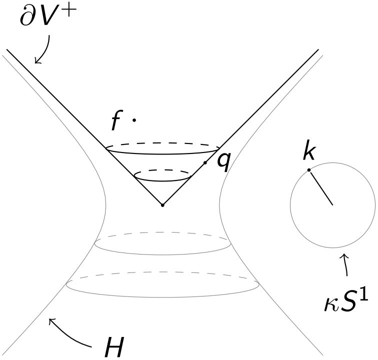

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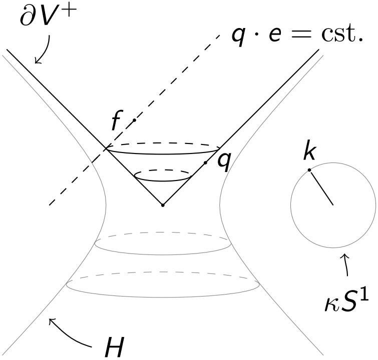

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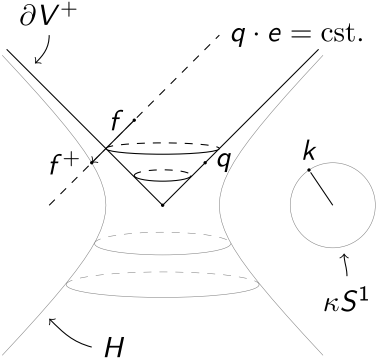
Step 2

- $A = [0, \bar{\mathrm{f}}/f_+] \in G_q$
- $\Rightarrow R(A, q) = A$
- $\mathrm{e}^{-\mathrm{i}k \cdot \frac{\mathrm{f}}{f_+}} u_1(q, f) = u_1\left(q, f^+\right)$



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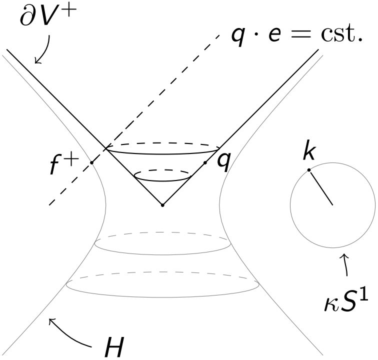

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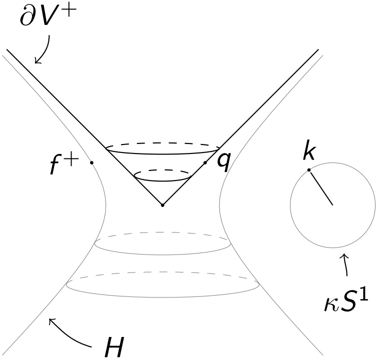

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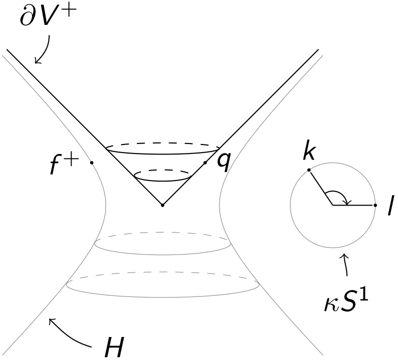
Step 3

$A = [\varphi, 0]$: q and f^+ invariant,
$F_1(f_+/2) := u_1(q, f^+)(k)$



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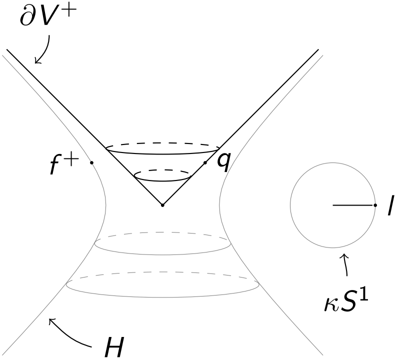

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- Substitution of the intertwiner equations yields the first part

$$u_1(p,e)(k) = \mathrm{e}^{\mathrm{i}k\cdot\frac{\mathrm{e}-\frac{e_-}{p_-}\mathrm{p}}{2p\cdot e}} F_1(p\cdot e).$$

- $2p\cdot e$ in exponent produces essential singularities at the boundary $\Im(e) = 0$.
- At any singularity one can show $\left|k\cdot\left(\mathrm{e}-\frac{e_-}{p_-}\mathrm{p}\right)\right| \leq \kappa$.
- u_1 is therefore an intertwiner iff F_{1r} in

$$F_1(p\cdot e) = \mathrm{e}^{-\mathrm{i}\frac{\kappa}{2p\cdot e}} F_{1r}(p\cdot e)$$

  is pol. bounded distributional boundary value of analytic function on H^+.
- $F_{1r}(p\cdot e) := 1$ yields the candidate

$$u_1(p,e)(k) = \mathrm{e}^{\mathrm{i}\frac{k\cdot\left(\mathrm{e}-\frac{e_-}{p_-}\mathrm{p}\right)-\kappa}{2p\cdot e}}.$$

Two-particle scalar intertwiners

- Similar result for the two-particle intertwiner u_2:

Lemma

Let $u_2(p, \tilde{p})(k, \tilde{k})$ the function given in *assumption 2*, which is a solution of

$$D(R(A,p)) \otimes D(R(A,\tilde{p}))u_2(p\Lambda(A), \tilde{p}\Lambda(A)) = u_2(p,\tilde{p})$$

Then there is a L^2_{loc}-function $F_2 \; : \; \mathbb{R}^2 \to \mathbb{C}$ such, that

$$u_2(p,\tilde{p})(k,\tilde{k}) = \mathrm{e}^{-\mathrm{i}k\cdot \frac{1}{\mathrm{\bar{p}}-\bar{\tilde{\mathrm{p}}}\frac{p_-}{\tilde{p}_-}}} \mathrm{e}^{-\mathrm{i}\tilde{k}\cdot \frac{1}{\bar{\tilde{\mathrm{p}}}-\mathrm{\bar{p}}\frac{\tilde{p}_-}{p_-}}} F_2\left((k\tilde{k})^{-1} \left(\mathrm{p} - \tilde{\mathrm{p}}\frac{p_-}{\tilde{p}_-} \right) \left(\tilde{\mathrm{p}} - \mathrm{p}\frac{\tilde{p}_-}{p_-} \right) \right)$$

- Extension of the characterization for u_2 to the coefficient functions u_{2c}, u_0 and u_{0c}:

Lemma

There are L^2_{loc}-functions F_0 and F_{0c}, such that the following equations hold:

$$u_{2c}(p,\tilde{p})(k,\tilde{k}) = \frac{\mathrm{e}^{+\mathrm{i}k\cdot\frac{1}{\bar{\mathrm{p}}-\bar{\tilde{\mathrm{p}}}\frac{p_-}{\tilde{p}_-}}}\,\mathrm{e}^{+\mathrm{i}\tilde{k}\cdot\frac{1}{\bar{\tilde{\mathrm{p}}}-\bar{\mathrm{p}}\frac{\tilde{p}_-}{p_-}}}}{F_2\left((k\tilde{k})^{-1}\left(\mathrm{p}-\tilde{\mathrm{p}}\frac{p_-}{\tilde{p}_-}\right)\left(\tilde{\mathrm{p}}-\mathrm{p}\frac{\tilde{p}_-}{p_-}\right)\right)}$$

$$u_0(p,\tilde{p})(k,\tilde{k}) = \mathrm{e}^{-\mathrm{i}k\cdot\frac{1}{\bar{\mathrm{p}}-\bar{\tilde{\mathrm{p}}}\frac{p_-}{\tilde{p}_-}}}\,\mathrm{e}^{+\mathrm{i}\tilde{k}\cdot\frac{1}{\bar{\tilde{\mathrm{p}}}-\bar{\mathrm{p}}\frac{\tilde{p}_-}{p_-}}}\,F_0(\dots)$$

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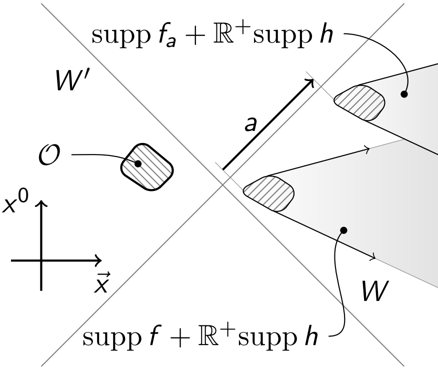
- Consider the function

$$\gamma(a) = \langle \phi, [B(g), \Phi(f_a, h)]\Omega \rangle \text{ , where } f_s := (\mathbf{1}, sn)_* f$$

Proof strategy:

- γ evaluates nontrivial matrix elements
- B tempered distribution $\Rightarrow$ pol. bounded
- rel. locality to Φ $\Rightarrow$ half-sided support



- dist. FT of γ is $\mathcal{S}'$-BV of an analytic function
- incompatible with singularities in u_2, u_0, ...

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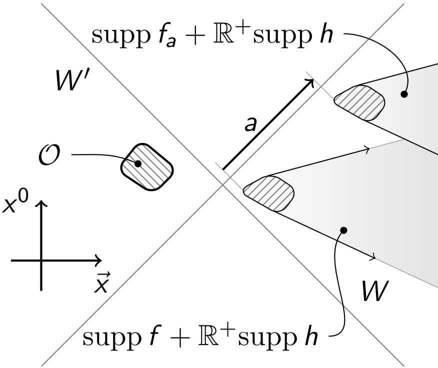

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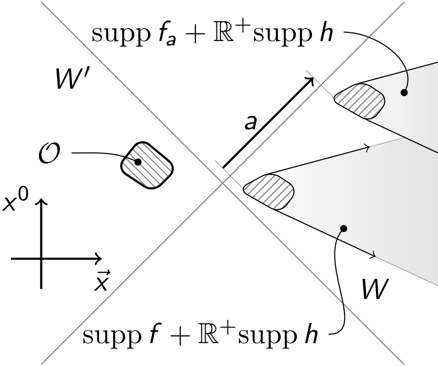

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Lemma (regularity of γ)

The function γ has the following properties:

1. **Support:** $\operatorname{supp}\gamma \subseteq (-\infty, -b]$
2. **Boundedness:** There are constants $C, L > 0$ and $N \in \mathbb{N}$, such that
$$|\gamma(a)| \leq C \left(\frac{1}{L} \chi_{[-L,0]-b}(a) + |a+b|^{N-1} \right) \; \forall a < -b.$$
3. **Continuity:** γ is a continuous function.

Lemma (holomorphic FT)

The holomorphic Fourier transform of a continuous polynomially bounded function $\gamma : \mathbb{R} \to \mathbb{C}$ with $\operatorname{supp} \gamma \subseteq (-\infty, -b]$ for some $b > 0$, which is defined by

$$\hat{\gamma}(z) = \int \mathrm{d}a \, \mathrm{e}^{-\mathrm{i}za} \gamma(a) \; \forall \, z \in H^+,$$

where $H^+ := \{z \in \mathbb{C} : \Im(z) > 0\}$ is the upper half-plane, has the following properties:

1. **Analyticity:** $\hat{\gamma}$ is an analytic function on H^+.
2. **Boundedness:** There are constants $C > 0, N \in \mathbb{N}$, such that

$$|\hat{\gamma}(z)| \leq C \mathrm{e}^{-b\Im(z)} (1 + \Im(z)^{-N}) \; \forall \, z \in H^+$$

3. **Distributional boundary value:** ...

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Lemma (holomorphic FT, part II)

3. **Distributional boundary value:** The sequence of distributions $\hat{\gamma}_t \in \mathcal{S}'(\mathbb{R})$, given by the restrictions of $\hat{\gamma}$ to horizontal lines of constant imaginary part $t > 0$,

$$\hat{\gamma}_t \;:\; \mathcal{S}(\mathbb{R}) \mapsto \mathbb{C},\, \varphi \mapsto \int \mathrm{d}s\, \gamma(s + \mathrm{i}t)\varphi(s),$$

converges for $t \to 0$ to the distributional FT of γ,

$$\hat{\gamma} \;:\; \mathcal{S}(\mathbb{R}) \to \mathbb{C},\, \varphi \mapsto \int \mathrm{d}a\, \gamma(a)\hat{\varphi}(a)$$

$$\text{with } \hat{\varphi}(a) := \int \mathrm{d}s\, \mathrm{e}^{-\mathrm{i}sa}\varphi(s) \text{ the FT on } \mathcal{S}(\mathbb{R}),$$

in the sense of $\mathcal{S}'(\mathbb{R})$: $\lim_{t\to 0} \hat{\gamma}_t(\varphi) = \hat{\gamma}(\varphi)\; \forall\, \varphi \in \mathcal{S}(\mathbb{R})$

- $\gamma(a)$ can be stated in terms of functions of $p_- \in \mathbb{R}$
- $\Psi(p,k) := \hat{f}(p)\tilde{u}_1(p,h)(k)$ with

$$\tilde{u}_1(p,h)(k) := \begin{cases} u_1(p,h)(k) & \text{for } p \in \partial V^+ \\ \overline{u_{1c}(-p,h)(k)} & \text{for } p \in \partial V^- \end{cases}$$

- $I(p,\tilde{p},k,\tilde{k}) := \mathrm{e}^{+\mathrm{i}k\cdot\frac{1}{\bar{\mathrm{p}}-\bar{\tilde{\mathrm{p}}}\frac{p_-}{\tilde{p}_-}}}\mathrm{e}^{-\mathrm{i}\tilde{k}\cdot\frac{1}{\bar{\tilde{\mathrm{p}}}-\bar{\mathrm{p}}\frac{\tilde{p}_-}{p_-}}} S(p,\tilde{p},\psi)$ with

$$\begin{aligned} S(p,\tilde{p},\psi) := \Theta(p\tilde{p})[&\hat{g}(\tilde{p},-p)F_0(2p\tilde{p}\mathrm{e}^{\mathrm{i}\psi}/\kappa^2) \\ &+\hat{g}(-p,\tilde{p})F_{0c}(2p\tilde{p}\mathrm{e}^{\mathrm{i}\psi}/\kappa^2)] \\ +\Theta(-p\tilde{p})[&\hat{g}(\tilde{p},-p)F_2(2p\tilde{p}\mathrm{e}^{\mathrm{i}\psi}/\kappa^2) \\ &+\hat{g}(-p,\tilde{p})F_2(2p\tilde{p}\mathrm{e}^{\mathrm{i}\psi}/\kappa^2)], \end{aligned}$$

- coordinate ψ is stable under $k,\tilde{k} \mapsto \lambda k, \lambda^{-1}\tilde{k}$ for $\lambda \in \mathrm{SO}(2)$

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- coordinate ψ is stable under $k,\tilde{k} \mapsto \lambda k, \lambda^{-1}\tilde{k}$ for $\lambda \in \mathrm{SO}(2)$

- With abbreviation $q := (\mathrm{p}, \tilde{p}, k, \tilde{k})$ (measure μ), one obtains

$$\gamma(a) = \int \frac{\mathrm{d}p_-}{p_-} \mathrm{e}^{\mathrm{i}p_- a/2} \int \mathrm{d}\mu(q) \overline{\phi(\tilde{p}, \tilde{k})} \Psi(p, k) I(p, k, \tilde{p}, \tilde{k})$$

- Singularities contained in I can be exposed: replacing ϕ and ψ by

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$$\phi_{\tilde{p}_0, \tilde{k}_0, \epsilon}(\tilde{p}, \tilde{k}) := \frac{\chi_{B_\epsilon(\tilde{p}_0, \tilde{k}_0)}(\tilde{p}, \tilde{k})}{\mu(B_\epsilon(\tilde{p}_0, \tilde{k}_0))}$$
$\rightarrow$ valid choice for $\phi \in \mathcal{H}_1$

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$$\Psi_{\mathrm{p}_0, k_0, \epsilon} := \hat{f}\left(p_-, \frac{|\mathrm{p}|^2}{p_-}\right) \delta_{\mathrm{p}_0, \epsilon}(\mathrm{p}) \delta_{k_0, \epsilon}(k)$$
$\rightarrow$ Ψ is determined by $\Phi(f, h)$, limiting procedure necessary.

- Resulting sequence of functions denoted by $(\gamma_{q_0, \epsilon})_{\epsilon > 0}$.

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Let $\mathrm{p}_0 \in \mathbb{R}^2$, $k_0 \in \kappa S^1$ such, that $\mathrm{p}_0 \nparallel k_0$. For $\epsilon > 0$, consider the function

$$\Psi_{\mathrm{p}_0,k_0,\epsilon} : \partial V \times \kappa S^1 \to \mathbb{C}\,,\ (p,k) \mapsto \hat{f}\left(p_-, \frac{|\mathrm{p}|^2}{p_-}\right)\delta_{\mathrm{p}_0,\epsilon}(\mathrm{p})\delta_{k_0,\epsilon}(k).$$

There is a sequence of sets of finitely many functions

$$\left((f^i_{\epsilon,N}, h^i_{\epsilon,N}) \in \mathcal{S}(\mathbb{M}) \times \mathcal{D}(H),\ i = 1, ..., M_{\epsilon,N}\right)_{N\in\mathbb{N}}$$

which conserve the support properties of $\Phi(f,h)$, i.e.

$$\operatorname{supp} f^i_{\epsilon,N} \subset W,\ \operatorname{supp} h^i_{\epsilon,N} \subset W \cap H\ \forall\, i = 1, ..., M_{\epsilon,N}, N \in \mathbb{N},$$

which converge to $\Psi_{\mathrm{p}_0,k_0,\epsilon}$ in the sense of L^2 up to a continuous function $c(\mathrm{p},k)$:
$\int \frac{\mathrm{d}p_-}{|p_-|}\mathrm{d}^2\mathrm{p} \int \mathrm{d}\nu(k) \left|\sum_{i=1}^{M_{\epsilon,N}} \hat{f}^i_{\epsilon,N}(p)\tilde{u}_1(p,h^i_{\epsilon,N})(k) - c(\mathrm{p},k)\Psi_{\mathrm{p}_0,k_0,\epsilon}(p,k)\right|^2$
converges to 0. The function c is has the property $c(\mathrm{p},k_0) = 1$.

Let $\mathrm{p}_0 \in \mathbb{R}^2$, $k_0 \in \kappa S^1$ such, that $\mathrm{p}_0 \nparallel k_0$. For $\epsilon > 0$, consider the function

$$\Psi_{\mathrm{p}_0,k_0,\epsilon} : \partial V \times \kappa S^1 \to \mathbb{C}\,,\ (p,k) \mapsto \hat{f}\left(p_-, \frac{|\mathrm{p}|^2}{p_-}\right)\delta_{\mathrm{p}_0,\epsilon}(\mathrm{p})\delta_{k_0,\epsilon}(k).$$

There is a sequence of sets of finitely many functions

$$\left((f^i_{\epsilon,N}, h^i_{\epsilon,N}) \in \mathcal{S}(\mathbb{M}) \times \mathcal{D}(H),\ i = 1, \ldots, M_{\epsilon,N}\right)_{N\in\mathbb{N}}$$

which conserve the support properties of $\Phi(f,h)$, i.e.

$$\mathrm{supp}\, f^i_{\epsilon,N} \subset W,\ \mathrm{supp}\, h^i_{\epsilon,N} \subset W \cap H\ \forall\, i = 1, \ldots, M_{\epsilon,N}, N \in \mathbb{N},$$

which converge to $\Psi_{\mathrm{p}_0,k_0,\epsilon}$ in the sense of L^2 up to a continuous function $c(\mathrm{p},k)$:

$$\int \frac{\mathrm{d}p_-}{|p_-|}\mathrm{d}^2\mathrm{p} \int \mathrm{d}\nu(k) \left|\sum_{i=1}^{M_{\epsilon,N}} \hat{f}^i_{\epsilon,N}(p)\tilde{u}_1(p, h^i_{\epsilon,N})(k) - c(\mathrm{p},k)\Psi_{\mathrm{p}_0,k_0,\epsilon}(p,k)\right|^2$$

converges to 0. The function c is has the property $c(\mathrm{p},k_0) = 1$.

- The analyticity of each $\hat{\gamma}_{q_0,\epsilon}$ is preserved in the limit $\epsilon \to 0$:

Lemma (compact convergence)

The set of sequences of functions

$$\hat{\gamma}_{q_0,\epsilon} \;:\; H^+ \to \mathbb{C},\, z \mapsto \int \mathrm{d}a\, \mathrm{e}^{-\mathrm{i}za} \gamma_{q_0,\epsilon}(a)$$

has the following property: For μ-almost all q_0 $\exists$ analytic function $\hat{\gamma}_{q_0}$ on H^+ such, that

$$\lim_{\epsilon \to 0} \hat{\gamma}_{q_0,\epsilon}(z) = \hat{\gamma}_{q_0}(z)\ \forall\, z \in H^+$$

in the sense of compact convergence.

- Consider the difference $\hat{\gamma}(z) := \hat{\gamma}_{q_1}(z) - P(z, q_1, q_0)\hat{\gamma}_{q_0}(z)$, with $q_0 \mapsto q_1$ by $(k_0, \tilde{k}_0) \mapsto (\lambda k_0, \lambda^{-1}\tilde{k}_0)$, P relative phase

Lemma (Uniform convergence)

Let $(\gamma_\epsilon)_{\epsilon>0}$ a sequence of analytic functions on H^+ with the following properties:

1. $\boxed{\lim_{\epsilon\to 0}\gamma_\epsilon = \gamma}$ exists in the sense of compact convergence, with γ an analytic function on H^+.
   The sequence fulfils the uniform bound
   $|\gamma_\epsilon(z)| < C\Im(z)^{-1}\,\forall z \in H^+, \epsilon > 0$ for some $C > 0$.
2. For $\epsilon > 0$, the (boundary-)$\boxed{\lim_{t\searrow 0}\gamma_\epsilon(\cdot + \mathrm{i}t) = g_\epsilon}$ exists and is given by a function $g_\epsilon \in L^1(\mathbb{R})$, where convergence is understood in the weak-* topology.
3. The corresponding sequence of boundary functions $(g_\epsilon)_{\epsilon>0}$ fulfils $\boxed{\lim_{\epsilon\to 0} g_\epsilon = 0 \text{ in } L^1(\mathbb{R})}$.

...

Lemma (Uniform convergence, part II)

$$
\begin{array}{ccc}
\gamma_\epsilon(\cdot + \mathrm{i}t) & \xrightarrow{\;t \searrow 0,\ \text{weak-*}\;} & g_\epsilon \\
{\scriptstyle \epsilon \to 0}\Big\downarrow {\scriptstyle +\text{uniform bound}} & & {\scriptstyle \epsilon \to 0}\Big\downarrow {\scriptstyle L^1} \\
\gamma(\cdot + \mathrm{i}t) & \overset{t \searrow 0,\ \text{weak-*}}{\dashrightarrow} & 0
\end{array}
$$

- Then $\gamma = 0$ on all of H^+. (using [SW '64, Thm. 2-17])

- $\Rightarrow \hat{\gamma}_{q_1}$ has a singularity, which is a contradiction!

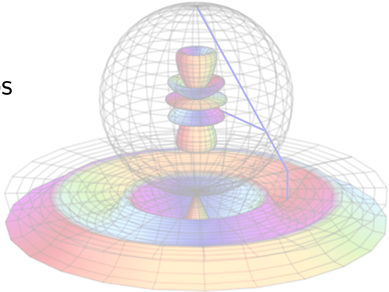
1 Introduction

2 Compact Localization

3 No-Go Theorem

4 Limit of Representations
- Reference Momenta & Little Groups
- Little Group Representations
- Construction of Intertwiners

5 Summary & Outlook



Pauli-Lubanski spin-vector

$$S^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\kappa} M_{\nu\lambda} P_\kappa$$

$M_{\nu\lambda}$: Lie-Algebra of generators of $\mathcal{L}_+^\uparrow$

- $m > 0$ interpretation: "angular momentum" in particle's rest frame
- $S^2 = S^\mu{}_\mu$ defines another Casimir operator.

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Comparison of the massive and massless case

Important distinction between massive and massless case:

m^2	$\mathrm{sp}\, P$	q	B_p	G_q	$\mathcal{H}_q$	$-\frac{1}{4}S^2$
1	H^+	$(1,\vec{0})$	$\sqrt{\widetilde{p}/m}$	$\mathrm{SU}(2)$	$\mathbb{C}^{2s}$	$m^2 s(s+1)$
0	∂V^+	$\frac{(1,\vec{e})}{2}$	$\frac{1}{\sqrt{p_-}}\begin{pmatrix} p_- & \overline{\mathrm{p}} \\ & 1 \end{pmatrix}$	$\widetilde{E(2)}$	$L^2(S^1)$	κ^2

- Construction of the previous objects is usually done separately for $m > 0$ and $m = 0$.
- Fundamentally different properties in the case $m = 0, \kappa > 0$
- How do these difficulties arise in the limit $\kappa = \mathrm{const.}$, $m \to 0$?
- Idea: Comparison between massive and massless fields is simplified, if construction is unified.

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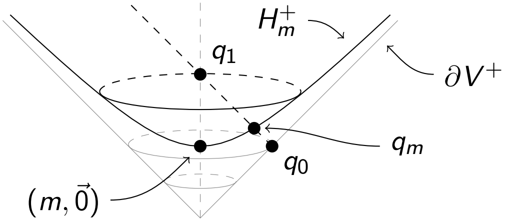
m-parametrized approach

- Reference momentum q_m is given by



$$\widetilde{q}_m = \begin{pmatrix} 1 & \\ & m^2 \end{pmatrix}$$

with q_{m-} independent of m.

- Usual choice for q is $(m, \vec{0})$, switching between conventions amounts to the Lorentz transform:

$$B_m := \begin{pmatrix} \sqrt{m} & \\ & \sqrt{m}^{-1} \end{pmatrix}, \text{ since } q_m \Lambda(B_m) = (m, \vec{0}).$$

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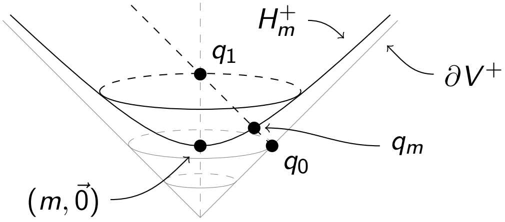

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m-dependence of Wigner rotations

- Massless form of the Wigner boost B_p is still valid for all m, $q_m\Lambda(B_p) = p \; \forall \; p \in H_m^+$, result depends on m only via q_m.
- Wigner rotation in m-parametrized form:

$$R = \underbrace{B_p A}_{=:C} B^{-1}_{p\Lambda(A)} = C B^{-1}_{q_m \Lambda(C)}, \; C =: \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

with C independent of m. Explicit form:

$$R = \frac{1}{\sqrt{|a|^2 + m^2|c|^2}} \begin{pmatrix} a & -m^2\overline{c} \\ c & \overline{a} \end{pmatrix} \begin{cases} \in \mathrm{SU}(2) & m = 1 \\ \in \widetilde{E(2)} & m = 0 \end{cases}$$

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Special cases

- For $m = 1$, $G_1 = \mathrm{SU}(2)$, there is a correspondence between R rotating the sphere and R acting as **Möbius transform** on the complex plane - stereographic projection.

$$[D(R)f](z) = f(R^{-1}.z) \text{ where } \begin{pmatrix} a & b \\ c & d \end{pmatrix}.z = \frac{az + c}{bz + d}$$

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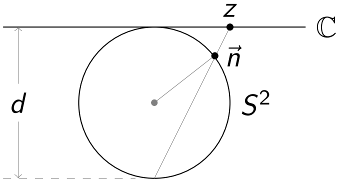
- Stereographic projection: identification between $z \in \mathbb{C}$ and $\vec{n} \in S^2$ given by

$$n_3 = \frac{d^2 - |z|^2}{d^2 + |z|^2},\ n_1 + \mathrm{i}n_2 = \frac{2zd}{d^2 + |z|^2}$$



- R corresponding to the usual choice $(m, \vec{0})$ can be obtained by conjugation with B_m:

$$R_m := B_m^{-1} R B_m = \frac{1}{\sqrt{|a|^2 + m^2|c|^2}} \begin{pmatrix} a & -m\bar{c} \\ mc & \bar{a} \end{pmatrix} \in \mathrm{SU}(2)$$

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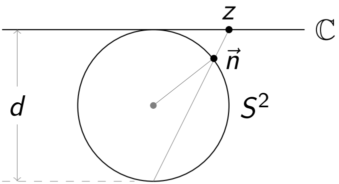

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- Representation spaces $\mathbb{C}^{2l+1}$ of $\mathrm{SU}(2)$ are spanned by spherical harmonics $Y_h^l(\vec{n}(z)) = \mathrm{e}^{\mathrm{i}h\arg z} P_h^l(n_3(z))$ with

$$\left(\frac{\mathrm{d}}{\mathrm{d}n_3}(1-n_3^2)\frac{\mathrm{d}}{\mathrm{d}n_3} + l(l+1) - \frac{h^2}{1-n_3^2}\right) P_h^l(n_3) = 0.$$

  (Legendre polynomials)

- Stereographic projection transforms the equation into

$$\left(\left(|z|\frac{\mathrm{d}}{\mathrm{d}|z|}\right)^2 + \frac{\kappa^2|z|^2}{\left(1+\left(\frac{|z|}{d}\right)^2\right)^2} - h^2\right) P_h^l(n_3(|z|)) = 0\ ,$$

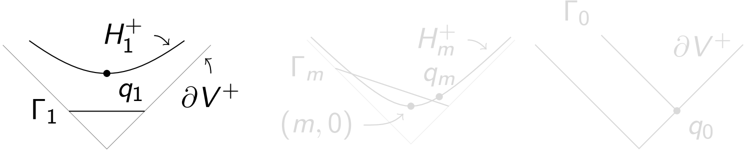
  with $\kappa^2 := 4l(l+1)/d^2$.

- Solutions $J_h(\kappa|z|)$ in the limit $d \to \infty$, $\kappa = \mathrm{const}$ span representation spaces $L^2(\kappa S^1)$ of $\widetilde{E(2)}$: (Bessel functions)

- Once m is chosen, one can construct the following parametrization of Γ_{q_m}:

$$\xi_d : \mathbb{R}^2 \to \Gamma_q,\ [\xi_d(z)]\tilde{} = \frac{d^2}{d^2 + |z|^2} \begin{pmatrix} |z|^2 & \bar{z} \\ z & 1 \end{pmatrix}$$

Crucial property: $\xi_d(R.z) = \xi_d(z)\Lambda(R)$



- Parametrization can also be given in terms of the usual choice for $m = 1$:

$$[\xi(z)]\tilde{} = (B_m^{-1})^\dagger(\mathbf{1} + \vec{\sigma} \cdot \vec{n})B_m^{-1}$$

Intuition: Lorentz-boosted "celestial sphere"

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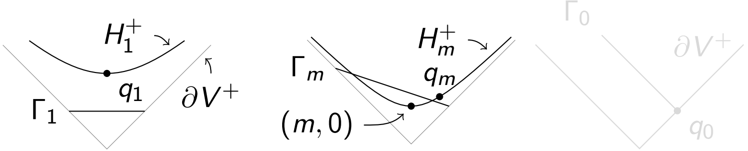

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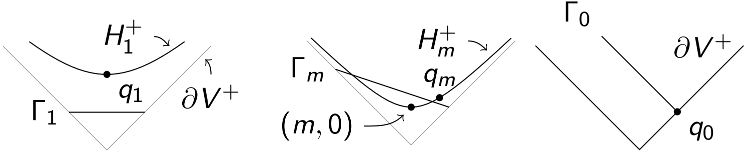

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Parametrization of string-localized intertwiners

- Therefore, the intertwiner $u : H_m^+ \times H \to \mathcal{H}_q$ defined by

$$u(p,e)(h) := \int \mathrm{d}^2 z \left(\frac{d^2}{d^2 + |z|^2} \right)^2 Y_h^l(\vec{n}(z))\, F(\xi_d(z)\Lambda(B_p)e),$$

  where F is a numerical function, inherits the desired covariance properties from Y_h^l.

- Infinite spin limit: $(d, l \to \infty,\ m \to 0,\ \kappa$ fixed$)$

$$u(p,e)(h) = \int \mathrm{d}^2 z\, \mathrm{e}^{\mathrm{i}h \arg z} J_h(\kappa|z|) F(\xi(z)\Lambda(B_p)e)$$
$$= \frac{\mathrm{i}^n}{2\pi} \int \mathrm{d}\varphi\, \mathrm{e}^{\mathrm{i}h\varphi} \int \mathrm{d}^2 z\, \mathrm{e}^{\mathrm{i}k(\varphi)\cdot z} F(\xi(z)\Lambda(B_p)e)$$

$$k(\varphi) := \kappa(\cos\varphi, \sin\varphi)$$

1 Introduction

2 Compact Localization

3 No-Go Theorem

4 Limit of Representations

5 Summary & Outlook
- Current form of the No-Go Theorem
- Characterization of Standard Subspaces
- Towards weaker Regularity Assumptions

Summary

- Infinite spin representations are known to imply weaker localization properties.
- Known quantum fields are localized in semiinfinite strings/cones.
- Compact (modular) localization is possible for two-particle wavefunctions.
- $\rightarrow$ Corresponding nontrivial operators do not exist.
- Result is based on the incompatibility between the analyticity of the relative commutator versus the singularities arising from the infinite spin covariance.

- First requirement to be weakened is that u_2 is an intertwiner.
- Any different class $\tilde{B}$ of operators localized in $\mathcal{O}$ has to generate vectors $B\Omega \in K(\mathcal{O})$.
- Can these be fundamentally different from the mentioned vectors $B(f)\Omega$?

Characterization of modular subspaces [Lechner, Longo '14]

In the one-particle Hilbert space of a 1d massless chiral/2d massive particle, modular subspaces corresponding to intervals/double cones can be characterized by the support of the inverse FT/momentum space analyticity.

- Application to present context needs several generalizations:
  - $d > 2$ requires intersection of infinitely many wedges.
  - behaviour of non-scalar representations
  - n-particle subspaces for $\mathcal{O} \subsetneq W$ are not necessarily tensor products of the one-particle subspaces.

- L^2_{loc} integrability of u_2, u_{2c}, u_0, u_{0c} is a technical assumption
- Idea: Apply the Schwartz Kernel Theorem and study $B(g)$ in terms of a distributional integral kernel
- Restrict distribution to cones using approximation technique for ψ
- cone-localized distributions can be understood as derivatives of continuous functions

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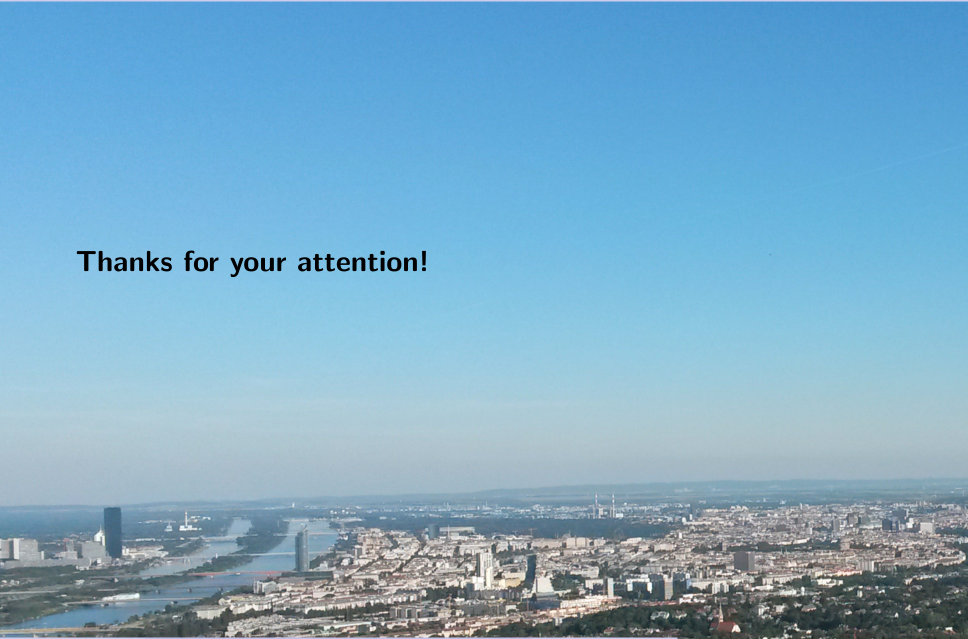

Thanks for your attention!