# String-localized Quantum Field Theory*

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# 1 What is string-localized quantum field theory?

String-localized quantum field theory (SQFT) has its historical roots in Quantum Electrodynamics (QED), addressing its gauge redundancy [Jordan (1935), Dirac (1955), Mandelstam (1962)] and its inherently non-local features related to the global Gauss Law [Ferrari et al. (1974), Buchholz (1982), Steinmann (1984)], see the short discussion in Sect. 2. Relatively recently, it has developped into a (mainly perturbative at the time of this writing) approach to interacting quantum field theory, whose main intention is to "liberate" the interactions of the Standard Model from unphysical degrees of freedom, that are brought in by gauge invariance of classical Lagrangians. A quote from [Schwartz (2014), Chap. 8.6] highlights the conceptual shortcomings of "gauge invariance" as a physical principle:

> *"Gauge invariance is not physical. It is not observable and is not a symmetry of nature* [... *It*] *is merely a redundancy of description we introduce to be able to describe the theory with a local Lagrangian."*

This does not belittle its success as a heuristic principle; but it calls for a better way to look at it.

In SQFT, "canonical quantization" is replaced by the direct construction of free quantum fields from unitary representations of the Poincaré group. All the notorious problematic issues with the canonical quantization of fields of spin or helicity $\geq 1$, can be avoided with the help of "string-localized" quantum fields. Their weaker localization than usual allows to construct massless vector potentials on the physical Hilbert space of the field tensor. In the massive case, it tames the bad UV-behaviour of local vector fields, thus allowing renormalizable interactions (at least in the power-counting sense, see Sect. 4).

This should be compared with the standard treatment of gauge theories using vector potentials defined on indefinite-metric state spaces, and with the treatment of weak interactions by first replacing the massive vector bosons by massless gauge bosons, and then making them massive via the Higgs mechanism.

SQFT does not refer to gauge invariance and canonical quantization. Instead, there arise constraints on the structure of admissible interactions by the condition that observable quantities must be independent of the auxiliary string. This condition has the same (in a few cases even superior) predictive power for the structure of interactions between Standard Model particles, as the usually invoked "gauge principle".

Being based on the fundamental principle that quantum theory should be defined on a Hilbert space, SQFT is a truly "autonomous" and gauge-free quantum approach that deduces quantum interactions from quantum principles. It also points the way in which the traditional axiomatics for interacting quantum fields [Streater and Wightman (1964)] must be extended in order to encompass the needs of the Standard Model. See Sect. 3.

The general idea applies as well to self-interactions of massless particles of helicity 2 ("gravitons") and their couplings to matter [Gass et al. (2023)]. See Sect. 4.6.

**Basic definitions of free string-localized quantum fields.** The starting formula for every model of SQFT is the definition of the free field

$$A_\mu(x,e) := \int_0^\infty ds\, F_{\mu\nu}(x+se)e^\nu. \tag{1}$$

Here, $F_{\mu\nu}$ is a massless or massive field strength tensor, defined on the "Wigner Hilbert space": the Fock space over the Wigner representation of helicity $h = \pm 1$ or spin $s = 1$, respectively [Weinberg (1964)]. $F_{\mu\nu}(x)$ is a local quantum field with precisely the physical degrees of freedom of massless or massive particles of spin/helicity 1.

In Eq. (1), $e$ is an arbitrary spacelike four-vector, that can be normalized as $e^2 = -1$. (The Lorentz metric convention is $\eta_{00} = +1$.) By its distributive nature, $A_\mu(x,e)$ should be smeared with a smooth function $c(e)$ of arbitrarily narrow support. Then $A_\mu(x,c)$ is regarded as a distribution in $x$. By its definition, $A_\mu(x,c)$ is localized in the cone emerging from $x$, that is spanned by the directions $e$ in the support of $c$. This cone is called a "string". By the locality of $F_{\mu\nu}$, it follows that two such fields commute with each other whenever their strings are spacelike separated from each other.

The crucial property of the free string-localized field Eq. (1) is

$$\partial_\mu A_\nu(x,e) - \partial_\nu A_\mu(x,e) = F_{\mu\nu}(x). \tag{2}$$

$A_\mu(x,e)$ is therefore a potential for the observable field tensor chosen in a particular gauge (an axial gauge). The distinction from "axial gauge quantization" is, however, that $F_{\mu\nu}(x)$ is already a quantum field on a Hilbert space, and $A_\mu(x,e)$ is just a "function" of it. Clearly, Eq. (2) holds as well for $A_\mu(x,c)$ smeared with a test function $c(e)$ of unit total weight.

If on the other hand, $F_{\mu\nu}(x)$ in Eq. (1) is taken as the exterior derivative of a canonically quantized local gauge potential $A_\mu(x)$ defined on a state space with indefinite inner product (Krein space), then $A_\mu(x,c)$ will be defined on the same Krein space, and differ from the former by the derivative of an "escort field":

$$A_\mu(x,c) = A_\mu(x) + \partial_\mu\phi(x,c). \tag{3}$$

Here, $\phi(x,c)$ is the smearing with $c(e)$ of the distribution

$$\phi(x,e) := \int_0^\infty ds\, A_\mu(x+se)e^\mu = \int_{C_{x,e}} A_\mu(y)\, dy^\mu, \tag{4}$$

where the integral is along the curve $C_{x,e}$ from $x$ to infinity in the direction $e$. As explained, this Krein space interpretation of Eq. (1) is not quite in the vein of SQFT, but it will allow for a novel understanding of infrared features of QED. See Sect. 4.1.

Eq. (3) is not considered as a gauge "transformation". It is rather a consequence of the definition, and marks the formal similarity with gauge theories: it explains – at least superficially – why the condition of string-independence of observables in SQFT (see below and Sect. 3) gives similar results as gauge invariance as a postulate, see also Sect. 5.

In order to monitor the string-independence of relevant quantities, one varies the smearing function $c(e)$ by a function $\delta c(e)$ of weight zero. Under such variations, it holds

$$\delta_c\big(A_\mu(x,c)\big) = \partial_\mu w(x,\delta c), \tag{5}$$

where $w$ is another string-localized field on the Wigner Hilbert space. If embedded into the Krein space, it is $w(\delta c) = \delta_c(\phi(c))$.

In the massive case, the derivative part of the two-point function of the Proca field

$$\big\langle 0|B_\mu(x)B_\nu(y)|0\big\rangle = -\big(\eta_{\mu\nu} + m^{-2}\partial_\mu\partial_\nu\big)W_m(x-x'), \tag{6}$$

where $W_m$ is the standard massive scalar two-point function, gives it the UV scaling dimension 2. As a consequence, interactions with Dirac fields are power-counting non-renormalizable. Because the derivative term is annihilated by the exterior derivative, the field tensor $F_{\mu\nu} := \partial_\mu B_\nu - \partial_\nu B_\mu$ also has dimension 2. Then by the string integration, $A_\mu(x,c)$ has scaling dimension 1. This improvement of the UV behaviour renders interactions involving $A_\mu(x,c)$ rather than $B_\mu(x)$ power-counting renormalizable. See Sect. 4.5.

The splitting Eq. (3) of the string-localized potential also holds in the massive case, with $A_\mu(x+se)$ in Eq. (4) replaced by the Proca field $B_\mu(x+se)$. In contrast to the massless case, both fields $A_\mu(x,c)$ and $\phi(x,c)$ are defined on the Wigner Hilbert space of $B_\mu(x)$.

**String-independence.** The string-localized free field is used to set up the perturbation theory for interacting quantum fields, see Sect. 3. The aim is to construct an interacting QFT with a string-independent S-matrix and string-independent local fields, to be regarded as the local observables of the theory. It turns out that such fields exist, and the S-matrix is string-independent in their vacuum sector.

Typically, however, charged and/or fermionic fields, which are regarded as "unobservable" already as (anti-)local free fields, become string-dependent and string-localized. In contrast to the treatment in gauge theories, the latter are well-defined on the Hilbert space. See Sect. 3 for the general pattern how this occurs, and Sect. 4.1 for the example of QED, where the string-localization of the interacting Dirac field is a physical feature.

It should be emphasized that the use of string-localized free fields does not affect the particles, nor the S-matrix elements between asymptotic many-particle states in the vacuum sector. Free fields like Eq. (1) are "made of" the same creation and annihilation operators as the physical local free fields like Eq. (2). Their purpose is to write the interactions used for perturbation theory in a conceptually more satisfactory way. They do not appear in the resulting interacting quantum field theory, whose observable fields are string-independent. Charged fields "absorb" the string-localization of the free escort fields, and asymptotic charged states may not belong to a Wigner Hilbert space because of infrared effects. See Sect. 3

## 2 The origins of SQFT

String-localized fields were already considered by [Jordan (1935), Dirac (1955), Mandelstam (1962)] in their attempts to formulate QED in terms of gauge-invariant expressions of the form

$$\psi_f(x) = e^{ie\int dy\, f(x-y)^\mu A_\mu(y)} \cdot \psi(x) \tag{7}$$

with suitable distributions $f^\mu(x-y)$ so that the exponential absorbs the multiplicative gauge transformation of the Dirac field. Among the simplest are solutions supported along a "string" (i.e., $f(x-y)$ is the characteristic function of the curve $C_{x,e}$ in Eq. (4)), so that the exponent in Eq. (7) is the escort field $\phi(x,e)$. In these works, Eq. (7) was understood as a classical entity, guiding the way to a gauge-invariant quantization prescription without necessarily a separate quantization of the two factors.

With the exponential understood as a quantum operator in the indefinite Fock space of gauge theory, Eq. (7) was used by [Steinmann (1984)], in order to improve the perturbation theory of QED. [Faddeev and Kulish (1970)] had attached similar exponential factors to the charged scattering states, in order to absorb infrared singularities of QED.

In axiomatic treatments, the inherent "non-locality" of QED related to the global Gauss Law was pointed out by [Ferrari et al. (1974)]: charged fields should not commute with the total charge operator which can be represented as the electric flux at spacelike infinity. Not addressing charged fields altogether, one can indirectly analyze their putative localization properties relative to observable fields by studying the localization properties of charged states when tested with observables. For charged states of QED, [Buchholz (1982)] pointed out the effect of "photon clouds" extending to infinity, but of arbitrary shape, and [Buchholz and Fredenhagen (1982)] proved that also in theories with a mass gap, states carrying topological charges exhibit localization properties "as if" they were created by string-localized fields.

It was therefore quite clear for a long time that the axiomatization of quantum fields by [Streater and Wightman (1964)] does not apply to a large class of fields in physically relevant theories.

A new development was initiated by [Mund et al. (2006)], who introduced string-localized *free* quantum fields. These can be employed to formulate interaction densities, with benefits as discussed in Sect. 1. The connection with string-localized quantum fields like Eq. (7) will become clear by Eq. (18) in Sect. 3: The charged free fields "dynamically absorb" the string-localized free fields and become themselves string-localized. The case of QED exactly reproduces Eq. (7) as a quantum field, as used by [Steinmann (1984)].

The original motivation to study string-localized free quantum fields [Mund et al. (2006)] was different, though. The authors wanted to overcome the limitations on the existence of local and covariant free quantum fields associated with massless representations of the Poincaré group: on the one hand, for finite helicity $h$, such fields do not exist as tensors of rank $\leq s$ (in particular, there exists no local and covariant vector potential $A_\mu(x)$ on the Fock space of physical photons) [Weinberg (1964)]; and on the other hand, local fields for particles in the "infinite-spin" (or "unbounded-spin") representations do not exist at all [Yngvason (1970)].

The first no-go result is the reason why one is forced to work with indefinite-metric state spaces (Krein spaces) in all gauge theories, with all the trouble that brings along. The second no-go result is presumably mainly of academic interest because to all our knowledge particles of infinite spin do not exist in our universe. If, however, quantum field theory would admit quantum fields with unusual features that would out-wit the no-go result and at the same time prevent their recording in detectors, one could imagine new candidates for "Dark Matter" [Schroer (2017)].

The new insight as compared to [Yngvason (1970)] was the groundbreaking notion of "modular localization" of [Brunetti et al. (2002)]. Localization of quantum fields is an algebraic property (the vanishing or non-vanishing of commutators), and not a geometric property that could be tested by some "position operators" (which do not exist in QFT). [Brunetti et al. (2002)] proposed to consider on the one-particle space of a given Wigner representation subspaces of states localized in wedge-shaped regions (Poincaré transforms of the standard wedge $\{x \in \mathbb{R}^4 : x^1 > |x^0|\}$), which are characterized by their behaviour under Lorentz boosts and CPT operators associated with the wedge. These data are provided by representation theory, and the assumed behaviour of the states is motivated by Tomita-Takesaki modular theory [E4].

In a nutshell: Modular theory (under conditions that are fulfilled in QFT) assigns to a von Neumann algebra $M$ and a vector in a Hilbert space, a unitary one-parameter "modular group" and an anti-unitary "modular conjugation". The former acts by automorphisms on $M$, and the latter maps $M$ to its commutant $M'$. For the algebra $A(W)$ of observables localized in a wedge and the vacuum vector, these are the boost subgroup and the CPT operator associated with $W$ in the given representation of the Poincaré group, and for free fields this algebra generates a real linear subspace of the one-particle representation space.

"Modular localization" turns the picture around, by characterizing, in terms of the given representation, subspaces $H_W$ of the one-particle space, whose elements are states localized in a wedge. States localized in smaller regions than wedges belong to intersections of several $H_W$. These intersections can be computed in terms of analyticity properties of their momentum space wave functions, and [Brunetti et al. (2002)] found that there exist one-particle states localized in arbitrarily narrow spacelike cones, and this is the best possible localization for the infinite-spin representations. Therefore, the algebras $A(C)$ of free fields localized in such cones $C$ should be nontrivial. Indeed, the authors were able to construct such fields by admitting them to be localized (in a distributional sense) on rays $x + e\mathbb{R}_+$, as in Eq. (1). But unlike in Eq. (1), these fields are not realized as integrals over local fields (which do not exist by [Yngvason (1970)]). Also states containing infinite-spin particles cannot be better localized than in cones [Longo et al. (2016)], pointing to the absence of local "composite fields" acting in the infinite-spin Wigner space.

The inventors of SQFT [Mund et al. (2006)] (by the wondrous detour via the infinite-spin field) anticipated the usefulness of string-localized quantum fields, both in the massive and massless case, for perturbative quantum field theory in and beyond the context of the Standard Model: They open the way to formulate interactions of massless particles (of any helicity) on their physical Hilbert space without the use of gauge theory. In the massive case, string-localized fields have a milder UV behaviour than their local counterparts (e.g., the Proca field Eq. (6) of power-counting dimension 2), manifest in their dimension being 1 or $\frac{3}{2}$ (just like scalar and Dirac fields) irrespective of their spin.

In particular, the generally alleged necessity of the "Higgs mechanism" as an artifice to confer masses to massless particles – which are in conflict with gauge invariance – can be put on trial [Schroer (2016), Schroer (2019)].

# 3 $L$-$Q$-pairs and $L$-$V$-pairs

Perturbative QFT starts from free fields (and their propagators) and proceeds by choosing an interaction density ("Lagrangian" – a Wick polynomial in free fields). It is notheworthy that a "free Lagrangian" is only required for the purpose of "canonical quantization", and is dispensible if free quantum fields are constructed directly on the Wigner Hilbert space.

The prototype of interactions between bosons and fermions in the Standard Model are the "minimal interactions"

$$L(x) = g\, A_\mu(x) j^\mu(x) \tag{8}$$

where $j^\mu$ are conserved free Dirac currents. $g$ is a coupling constant. In the massles case, $A_\mu$ is only defined on a Krein space, and in the massive case (with the Proca field $B_\mu$ as in Eq. (6) in the place of $A_\mu$), $L$ is a non-renormalizable interaction.

If one replaces $A_\mu(x)$ in Eq. (8) by a string-localized potential $A_\mu(x,c)$, then the resulting interaction

$$L(x,c) = g\, A_\mu(x,c) j^\mu(x) \tag{9}$$

is defined on a Hilbert space and is power-counting renormalizable both in the massless and massive case.

The challenge is to establish that the resulting perturbative S-matrix does not depend on the arbitrary choice of the string $c$. To control the dependence of the interaction on $c$, one may take the string-variation:

$$\delta_c\big(A_\mu(c) j^\mu\big) = \partial_\mu w(\delta c) j^\mu = \partial_\mu\big(w(\delta c) j^\mu\big), \tag{10}$$

or (if the escort field $\phi$ is defined) split the interaction directly

$$A_\mu(c) j^\mu = A_\mu j^\mu + \partial_\mu \phi j^\mu = A_\mu j^\mu + \partial_\mu\big(\phi j^\mu\big) \tag{11}$$

into a local and a string-localized part. The crucial feature in both options is that the string-dependence is manifested as a total derivative. Total derivatives added to the Lagrangian do not affect classical equations of motion, but they do affect the quantum S-matrix

$$S = Te^{i\int dy\, L(y)}, \tag{12}$$

because derivatives in general do not commute with time-ordering. As a consequence, the S-matrix will in general suffer from "obstructions" against string-independence, and it may be necessary to add higher-order corrections (in the coupling constant $g$ and in the polynomial degree in the fields) to the interaction density which cancel the obstructions. This procedure *deduces* the same structure of the higher-order interactions that is usually ascribed to gauge invariance, but with an entirely different rationale.

In general, one will have

$$L(y,c) = g\,L_1(y,c) + \frac{g^2}{2}L_2(y,c) + \dots . \tag{13}$$

In a renormalizable theory, the series should stop after the second-order term, because the polynomial degree of the Wick polynomials $L_n$ would increase beyond the power-counting bound. That the obstructions can be cancelled in the first place, and that the series indeed stops *automatically* for all Standard Model interactions (studied to-date, see Sect. 4) is perhaps one of the most compelling facts about SQFT.

In first order, the S-matrix is just $ig\int dy\,L_1(y,c)$. It is string-independent if

$$\delta_c\big(L_1(y,c)\big) \overset{!}{=} \partial_\mu Q_1^\mu(y,c,\delta c), \tag{14}$$

with some four-vector $Q_1^\mu$ of Wick polynomials in free fields. This is in particular the case when

$$L_1(y,c) \overset{!}{=} L_1^{\mathrm{loc}}(y) + \partial_\mu V_1^\mu(y,c), \tag{15}$$

where $L_1^{\mathrm{loc}}(y)$ is a string-independent (point-localized) first-order interaction, and $V_1^\mu$ another four-vector of Wick polynomials in free fields.

The above Eq. (10) and Eq. (11) are special cases of Eq. (14) and Eq. (15). The data in Eq. (14) are called an "$L$-$Q$-pair". The data in Eq. (15) are called an "$L$-$V$-pair". Not every $L$-$Q$-pair arises from an $L$-$V$-pair on the same Hilbert space. An example is QED where $w$ in Eq. (10) is defined on the Wigner Hilbert space, while $\phi$ in Eq. (11) is only defined on the Krein space.

The condition of string-independence of the S-matrix has been worked out in a model-independent way in [Mund et al. (2023)]. (Some specific details there are too narrow to encompass theories like Yang-Mills or QCD, but the assumptions can be easily relaxed.) The outcome is, in a nutshell:

If one starts from an $L$-$Q$-pair, one obtains a recursive formula for higher-order interactions $L_n(c)$ which cancel the obstructions coming from all contributions $L_m(c)$ $(m < n)$ up to the derivative of some higher-order four-vector $Q_n^\mu$ (which vanishes upon integration over $y$). These cancellations occur only in the "adiabatic" limit where the spacetime cutoff of the coupling constant is removed.

If one starts from an $L$-$V$-pair, one obtains a recursive formula for string-dependent higher-order interactions $L_n(c)$ and derivative terms $\partial_\mu V_n^\mu(c)$, as well as string-independent higher-order interactions $L_n^{\mathrm{loc}}$, which cancel the obstructions coming from all contributions $L_m(c)$,

$L^{\rm loc}_m$, $V^\mu_m$ with $m < n$. This formulation is in fact much more powerful than the $L$-$Q$-pair formulation, because it allows exact cancellations even when the coupling constant $g$ is cut-off at large spacetime distances. It is therefore appropriate for the mathematically rigorous renormalization theory of [Epstein and Glaser (1973)] ("causal perturbation theory", [E1]) in which the S-matrix is expanded into a series of distributions in the cut-off function $g(x)$, and UV loop renormalization is an issue of the extension of these distributions to its UV-singular points.

Moreover, the $L$-$Q$-pair formulation just establishes the existence of a string-independent S-matrix, while the $L$-$V$-pair formulation allows to assert the equality (at tree-level) of the S-matrix obtained with the string-dependent interaction $L(c)$ and the S-matrix obtained with the string-independent interaction $L^{\rm loc}$. The latter is therefore also a tool to compare the results of SQFT with results obtained in local approaches. This applies even when $L^{\rm loc}_1$ is non-renormalizable (like the minimal coupling of massive vector bosons): The series $L_n(c)$ stops with the power-counting renormalizable $L_2(c)$, while the non-renormalizable series $L^{\rm loc}_n$ need not stop. (A notable exception is the Higgs potential Eq. (28), which stops with the quartic term.) The equivalence at tree-level is expected to be instrumental for the indirect renormalization of the non-renormalizable local perturbation series.

**The structure of obstructions.** By the very derivative structure of $L$-$Q$- or $L$-$V$-pairs and the recursion started by them, all obstructions arise because derivatives do not commute with time-ordering. They can be computed in terms of the quantities (where $Y^\mu$ and $X$ are Wick polynomials)

$$O_{Y(y)}(X(x)) := T\big(\partial_\mu Y^\mu(y)X(x)\big) - \partial^y_\mu T\big(Y^\mu(y)X(x)\big). \tag{16}$$

In contrast to ordinary perturbation theory in terms of Feynman propagators, or equivalenty in terms of retarded propagators, the obstructions contribute additional terms to the perturbative expansion which contain $\delta$-functions or string-integrations (similar as in Eq. (1)) over $\delta$-functions. It is essential that the obstructions are much better localized than all other (string-independent) terms in the perturbative expansion (completely delocalized integrals over propagators).

This feature explains, not least, why the obstructions can be cancelled by higher-order interactions. In the construction of interacting fields, it is also responsible for an interesting new "dichotomy" to be outlined next.

**Interacting quantum fields.** Interacting quantum fields are defined by "Bogoliubov's formula", i.e., formally the insertion of the free field into the S-matrix:

$$\Phi\big|_L(x) := \big(Te^{i\int dy\,L(y)}\big)^*\big(T\Phi_0(x)e^{i\int dy\,L(y)}\big). \tag{17}$$

This formula can be given a rigorous meaning in causal perturbation theory in terms of "relative S-matrices" [E1].

In gauge theories formulated on a Krein space, the interacting field is a priori a local (or anti-local) field on the Krein space. The BRST (or Gupta-Bleuler) method allows to pass from the Krein space to the physical Hilbert space, on which only BRST quantities are defined [Kugo and Ojima (1979)]. The interacting fields in general will not satisfy this condition. E.g., while the interacting Maxwell tensor and Dirac current of QED are defined on the Hilbert space, the anti-local interacting charged Dirac field is not (because its BRST variation is a ghost-valued gauge transformation). This means that the "final" theory has charged states, but no charged fields to create them from the vacuum. See also Sect. 5.

In SQFT, the interacting fields are constructed on the physical Hilbert space. But they will in general be string-dependent. String-dependent fields cannot be local observables, but their presence in the theory is a valuable tool, because they create states from the vacuum that cannot be reached by the observables because they belong to different superselection sectors, see Sect. 4.1, and also [E3].

As pointed out in Sect. 2, the traditional axiomatization of interacting quantum fields as in [Streater and Wightman (1964)] treating all fields in the same way, except admitting anti-locality for fermionic fields, is too narrow for theories with long-range interactions or with topological charges. In SQFT, one finds that many quantum fields (also bosonic ones) become string-localized under the interaction. Only fields that remain local can be regarded as observables of the interacting QFT. In QED, this is indeed the case for the Maxwell tensor and the Dirac current, while the interacting Dirac field is string-localized. See Sect. 4.1.

The $L$-$Q$-pair formulation allows a characterization of those free fields that will remain local under the interaction. The $L$-$V$-pair formulation offers a more powerful tool to understand also the string-localized interacting fields, by virtue of the formula [Mund et al. (2023)]

$$\Phi\big|_{L(c)}(x) = \big(\Phi_{[g]}\big)\big|_{L^{\mathrm{loc}}}(x). \tag{18}$$

Because $L(c)$ is not local, one has a priori no control about the localization of the interacting field on the left-hand side. Eq. (18) expresses it as a modified field $\Phi_{[g]}$, perturbed with the local interaction $L^{\mathrm{loc}}$. The modified field (also called "dressed field")

$$\Phi_{[g]} = \Phi_0 + g\Phi_{[1]} + \frac{g^2}{2}\Phi_{[2]} + \dots \tag{19}$$

has an expansion into Wick polynomials in possibly string-localized free fields, without retarded integrals. It therefore belongs to the (string-localized) Borchers class [Borchers (1960)] of the free fields (it is relatively local w.r.t. the free fields) and has itself no nontrivial S-matrix. The point is that the local interaction $L^{\mathrm{loc}}$ preserves the localization of the dressed field relative to the interacting observables, so that the right-hand side of Eq. (18) makes a statement about the localization of the interacting field on the left-hand side.

The dressed field is much easier to compute than the actual interacting field, e.g., Eq. (20). The computation is done in terms of obstructions (regarded as derivations on the free field algebra). If the dressed field is local, then so is the interacting field. This allows the identification of the observables of the interacting QFT without their actual computation.

# 4 Achievements in the Standard Model (and beyond)

We summarize several studies of SQFT applied to the Standard Model and to perturbative Quantum Gravity. The general vein of the results is that SQFT explains all the structures of interactions that one is used to attribute to gauge invariance. The main distinction is that the predictions of SQFT arise from the need of a renormalizable interaction on a Hilbert space, while "gauge invariance" is rather a practical device to deal with redundancy, elevated to an esthetical "principle".

The results presented are not (yet) established in complete mathematical rigour at all orders of perturbation theory. Most of them are at tree-level – to all orders a few, otherwise verified up to sufficiently high order so as to make them compelling all-order conjectures. The extension to loop level requires the microlocal renormalization scheme as in [Epstein and Glaser (1973)]. SQFT, with the exception of gravity, is power-counting renormalizable. This is true even with minimally coupled massive vector bosons, which is non-renormalizable in local QFT. In local QFT, the power-counting criterium implies that all renormalization constants can be "absorbed" into finite redefinitions of the masses and coupling constants of the theory. A similar result is expected for SQFT. The microlocal renormalization problem simplifies considerably when the string-smearing is done before the renormalization, to the extent that the singular points in configuration space that need loop renormalization are the same as in local QFT [Gass (2022)]. The freedom of renormalization has been discussed at tree-level in [Cardoso et al. (2018)]; but the more recent experience with all the models presented below indicates that the necessary string-independence can be achieved without expoiting the full freedom.

Precursors of the results to be presented (as far as S-matrices and the need for higher-order interactions are concerned) were achieved in the local setting of gauge theory (using causal perturbation theory). Many of them are assembled in [Scharf (2001)], see also [Aste et al. (1999)]. Also there, gauge invariance is not assumed, but explained as a consequence of the need to be able to descend from a Krein space to a Hilbert space. For this purpose, the consistency of the BRST construction was analyzed in higher orders, with similar results as the string-independence of S-matrices in SQFT. See Sect. 5.

## 4.1 QED

Read more in [Mund et al. (2020), Mund et al. (2022)].

At first sight, the easiest application of SQFT is QED with the $L$-$Q$-pair Eq. (10), where $j^\mu$ is the Dirac current. Namely, all obstructions of the S-matrix, computed according to the schemes of Sect. 3, vanish by virtue of the standard Ward identity. So, there is no need for higher-order interactions, and $L(c) = g\,A_\mu(c)j^\mu$ gives a string-independent S-matrix. However, in order to compare it with the S-matrix of QED in the standard approach, one must embed $L(c)$ into the Krein space, and consider the $L$-$V$-pair $A_\mu(c)j^\mu = A_\mu j^\mu + \partial_\mu(\phi j^\mu)$.[1] One can then establish the equivalence of the S-matrices in the vacuum sector.

The interesting part is the "dressed Dirac field". It turns out to be, order by order in perturbation theory,

$$\psi_{[g]}(x,c) = e^{ig\phi(x,c)}\psi(x), \tag{20}$$

where, according to Eq. (4), $\phi(x,c)$ is $\int_{C_{x,e}} A_\mu(y)\,dy^\mu$, smeared with $c(e)$. This expression looks identical with the classical field Eq. (7), that was considered by [Jordan (1935), Dirac (1955), Mandelstam (1962)] in their attempts to define the quantization of QED in terms of gauge-invariant quantities. But Eq. (20) appears in string-localized QED as a quantum field from the start, with the exponential field as a quantum operator. [Steinmann (1984)] has used it in the same way.

At this point, however, the infrared singularity of QED [E2] strikes. The massless escort field $\phi(c)$ is actually logarithmically divergent. But its exponential can be defined as a Weyl operator with an IR renormalization. As a consequence, charged states created by dressed Dirac fields with different string smearing functions $c$ must belong to different superselection sectors. Indeed, $c(e)$ can be identified with the directional profile of the asymptotic electric flux density ("photon cloud") evaluated in such states. Moreover, the energy spectrum of the charged states has a sharp lower bound at the electron mass, but it does not contain a sharp mass-shell: charged particles are infra-particles. These features of QED were anticipated by axiomatic arguments a long time ago [Ferrari et al. (1974), Fröhlich et al. (1979), Buchholz (1982), Buchholz (1986)]. SQFT confirms this insight by an actual Hilbert space construction.

The dressing factor also contributes a complex phase to the commutation relations of the Dirac field Eq. (20). These are more general than "anyonic" commutation relations, because the phase factor is a continuous function both of the strings and of the distance $x - x'$.

According to Eq. (18), the actual interacting Dirac field is the dressed Dirac field Eq. (20) subjected to the point-local interaction of QED. The well-known IR divergences of the standard QED amplitudes interfere with the IR structure of the exponential escort field to give a "dynamically deformed superselection structure", where the photon clouds depend on the momenta of other charged particles in a scattering process.

The string-localization of the interacting Dirac field is a most welcome feature. Namely, an anti-local field would have to commute with the asymptotic electric flux density, and hence with the total flux at infinity which – by Gauss' Law – should be the total charge operator. SQFT neatly solves this conflict.

It has been stressed that for similar reasons, related to the local Gauss Law, "longitudinal photon degrees of freedom" must be present in QED. In SQFT, they are present in the form of the exponentiated escort field, turning the electron into an infra-particle.

[1] To be precise, one must also eliminate the "null field" $\partial_\mu A^\mu$, because it has vanishing correlations with all observable fields but not with the escort field, see [Mund et al. (2022)].

An instructive variant is scalar QED. The $L$-$Q$-pair is again Eq. (10), but with the scalar current $j^\mu = -i(\varphi^* \partial^\mu \varphi - \partial^\mu \varphi^* \varphi)$. Here, there is a freedom to renormalize the propagator of the derivative fields

$$\big\langle 0|T\partial_\mu \varphi(x)\partial'_\nu \varphi(x')|0\big\rangle = \partial_\mu \partial'_\nu \big\langle 0|T\varphi(x)\varphi(x')|0\big\rangle - ic_r \eta_{\mu\nu}\delta(x-x').$$

The obstructions depend on the choice of the real renormalization parameter $c_r$. It turns out that one needs a second-order Lagrangian $\frac{1}{2}L_2(c) = (1-c_r)\cdot A_\mu(c)A^\mu(c)\varphi^*\varphi$, where gauge theory would have expected the coefficient 1. (This feature is also known from local approaches.) Although the interaction Lagrangian depends on $c_r$, the S-matrix does not [Tippner (2019)]. Gauge invariance plays no role.

## 4.2 Yang-Mills

Read more in [Gass et al. (2021)].

Yang-Mills theory starts in SQFT with the identification of an $L$-$Q$-pair for the self-interaction. It turns out that the most general cubic self-interaction of any number of free massless vector fields, of the form

$$L_1(c) = -\sum\nolimits_{a,b,c=1}^N f_{abc}\partial^\mu A_a^\nu(c) A_{b,\mu}(c) A_{c,\nu}(c), \tag{21}$$

must necessarily have totally anti-symmetric coefficients $f_{abc}$ in order to yield an $L$-$Q$-pair:

$$\delta_c\Big(-\frac{1}{2}\sum\nolimits_{a,b,c=1}^N f_{abc}F_a^{\mu\nu}A_{b,\mu}(c)A_{c,\nu}(c)\Big) = \partial_\mu\Big(\sum\nolimits_{a,b,c=1}^N F_a^{\mu\nu}A_{b,\nu}w_c(\delta c)\Big). \tag{22}$$

Computing the resulting obstructions against string-independence of the S-matrix in second order, one finds that these obstructions can only be cancelled if $f_{abc}$ satisfy the Jacobi identity, i.e., they are the structure constants of some $N$-dimensional Lie algebra. In this case,

$$L_2(c) = -\frac{1}{2}\sum\nolimits_{a,b,c,d,e=1}^N f_{abe}f_{cde}A_a^\mu(c)A_b^\mu(c)A_{c,\mu}(c)A_{d,\nu}(c), \tag{23}$$

and $L_n(c) = 0$ for $n > 2$. Thus, $L(c) = gL_1(c) + \frac{g^2}{2}L_2(c)$ is exactly the interaction part of the standard Yang-Mills Lagrangian, with the gauge potentials replaced by the string-localized potentials. The Lie algebra structure and the quartic Yang-Mills coupling are thus deduced from SQFT, rather than manifestations of an a priori assumed gauge invariance.

## 4.3 QCD

We report here some unpublished work in progress. For QCD in SQFT, one may add to Eq. (22) the obvious $L$-$Q$-pair of minimal interactions

$$\delta_c\big(A_{a,\mu}(c)j_a^\mu\big) = \partial_\mu\big(w_a(\delta c)j_a^\mu\big). \tag{24}$$

It turns out that there are no new obstructions, and $L_2(c)$ in Eq. (23) unchanged. Again, this reproduces the QCD interaction of gauge theory, with $A^a_\mu$ replaced by $A^a_\mu(c)$.

An interesting feature is called "lock-key scenario": One might consider the $L$-$Q$-pair Eq. (24) separately, without the self-interaction. It then turns out that the violation of the Ward identity for non-abelian currents $j^\mu_a = \overline{\psi}\gamma^\mu\tau_a\psi$ leads to an obstruction in second order, that cannot be cancelled by a quartic interaction $L_2(c)$. This is the "lock". On the other hand, including the self-interaction, the mixed contributions in second order precisely cancel this obstruction. This is the "key". Thus, minimal interactions of non-abelian quark currents require the Yang-Mills self-interaction.

The dressed quark field seems (in low orders) to be non-abelian version of Eq. (20) with a path-ordered exponential. This would comply with the idea that the dressed field formally corresponds to a classically gauge-invariant quantity.

## 4.4 Chirality of weak interactions

The chirality of weak interactions is a prediction from SQFT, that goes beyond predictions from gauge invariance. Here is a brief account. Read more in [Gracia-Bondia et al. (2018)].

One first fixes the physical field content: $W$- and $Z$-bosons with masses $M_Z > M_W > 0$, and the leptons (electrons and neutrinos) with $m_e > 0$ and $m_\nu$ arbitrary. One makes a most general ansatz for an $L$-$Q$-pair of minimal couplings of massive and massless vector bosons to chiral lepton fields. As in Sect. 4.3, Eq. (25) has to be supplemented by self-interactions among the boson fields, see also Sect. 4.5. The leptonic $L$-$Q$-pair has the form

$$L_1(c) = \sum\nolimits_{a=1}^4 \big(A_{a,\mu}(c)(J^\mu_a + J^{\mu 5}_a) + \phi_a(c)(S_a + S^5_a)\big). \tag{25}$$

The indices $a = 1, 2$ refer to the $W$-bosons, $a = 3$ to the $Z$-boson, and $a = 4$ to the photon. $\phi_a(c)$ $(a = 1, 2, 3)$ are the escort fields of the three massive vector bosons, and $\phi_4 = H$ is the (string-independent) scalar Higgs field. $J^\mu_a$ and $J^{\mu 5}_a$ are charged and neutral (axial) vector currents, and $S_a$ and $S^5_a$ are (pseudo)scalars. They involve arbitrary combinations of $\overline{\psi} \dots \psi$ with $\psi \in \{\psi_e, \psi_\nu\}$ with a priori undetermined coefficients. There is only one coupling constant, because the condition of string-independence confirms the standard formula for the electric coupling constant as a function of the Weinberg angle $\cos\theta := M_W/M_Z$. But the direction of the deduction is reversed: in the Glashow-Salam-Weinberg (GSW) model, the Weinberg angle and the mass ratio are determined as functions of the two coupling constants.

Already the $L$-$Q$-pair condition (string-independence in first order) fixes the coefficients within the scalars $S_a$ resp. pseudoscalars $S^5_a$ as mass multiples of the coefficients within the vector resp. axial currents. In particular the terms involving $S_4$ and $S^5_4$ can be identified with the Yukawa couplings of the GSW model. All other coefficients are then separately determined by the string-independence in second order.

The surprise is: One finds that the axial and vector couplings to the $W$-bosons can only differ by a factor $\varepsilon$ with $\varepsilon^2 = 1$. The charged weak coupling therefore necessarily exhibits maximal parity violation. With the choice $\varepsilon = -1$, this is the famous empirically known "$V - A$-structure" – but now as a *prediction* from SQFT. (The choice $\varepsilon = +1$ is equally possible – it is unitarily equivalent by the parity operator.) The coupling to the photon comes out non-chiral, and the non-maximal parity violation of the $Z$-couplings comes out as in the GSW model.

Along the way, it is *deduced* that the total currents $J_a^\mu + J_a^{\mu 5}$ can be written in the form

$$J_a^\mu + J_a^{\mu 5} = \overline{\psi}\gamma^\mu \pi(\tau_a)\psi \tag{26}$$

with the well-known chiral representation $\pi$ of $U(1) \times SU(2)$ of the GSW model.

Another result of the SQFT treatment of massive vector bosons is that the $L$-$Q$-pair *requires* a coupling to (at least) one scalar field of arbitrary mass: the Higgs field. [Gracia-Bondia et al. (2018)] do not exhibit the analysis of obstructions in the Higgs sector. However, the simpler case of the coupling of a single massive vector boson to a scalar field (with or without minimal interactions with a Dirac field) shows that the cancellation of obstructions requires the scalar field to have a potential of the precise shape of the famous double-well Higgs potential, as will be discussed next.

## 4.5 The Higgs potential

Read more in [Mund et al. (2023)].

The coupling of a single massive vector boson to a scalar field admits an $L$-$V$-pair

$$m\big(A^\mu B_\mu H + A^\mu \phi \partial_\mu H - \frac{m_H^2}{2}\phi^2 H\big) = m\, B^\mu B_\mu H + m\partial_\mu\big(B^\mu \phi H + \frac{1}{2}\phi^2 \partial_\mu H\big) \tag{27}$$

Here, $B_\mu(x)$ is the local Proca field of mass $m$, $A_\mu \equiv A_\mu(c)$ is the string-localized potential computed by Eq. (1) from the Proca field tensor $F_{\mu\nu}(x)$, and $\phi \equiv \phi(c)$ is the escort field, Eq. (4). $H$ is the scalar field of mass $m_H$. It will retrospectively deserve the name "Higgs field".

Eq. (27) is essentially unique, up to the addition of a salient term $aH^3$ on both sides, and more terms that have to vanish because they would produce obstructions that cannot be cancelled in second order. The (positive) masses $m$, $m_H$, and the coefficient $a$ of the scalar self-coupling are free parameters at first order.

Recall from Sect. 3 that the $L$-$V$-pair formalism allows the detailed identification of the string-independent S-matrix with interaction $L(x, c)$ with the S-matrix with a local interaction $L^{\mathrm{loc}}(x)$. The latter may be power-counting non-renormalizable (the original problem with massive vector bosons), but the model is expected to be renormalized by the identification of its perturbative series with the renormalizable string-localized series.

The main result is, that the cancellation of obstructions in second order requires unique higher couplings between the vector boson and the Higgs field – up to a quartic Higgs self-coupling

$bH^4$ with an arbitrary coefficient $b$, and in third order uniquely fixes $a$ and $b$. In particular, $gaH^3 + \frac{g^2}{2}bH^4$ is exactly the interaction part of the Higgs potential

$$\frac{1}{2}m_H^2 H^2(1 + \frac{g}{2m}H)^2, \tag{28}$$

familiar from the Higgs model of "spontaneously broken gauge symmetry".

But there is neither an "unbroken phase" (the positive mass of the vector boson was set from the start), nor a "mechanism" of breaking a "symmetry" that does not exist in a physical sense, the the quote in Sect. 1. Only the Higgs field and its potential are confirmed by SQFT.

The non-renormalizable local counterpart interaction $L^{\text{loc}}(x)$ can also be computed in the model. It coincides, up to a renormalization, with the classical interaction Lagrangian that one would expect from the "Higgs mechanism".

### 4.6 Gravity

Read more in [Gass et al. (2023)].

The construction Eq. (1) of a string-localized potential for the Maxwell field can be generalized to a string-localized tensor potential $h_{\mu\nu}(x, c)$ for the local field $R_{[\mu\kappa][\nu\lambda]}(x)$ of helicity $h = \pm 2$ on its Wigner Hilbert space. The latter can be interpreted as the (linearized) Riemann tensor of perturbative Quantum Gravity, and the potential is the string-localized counterpart of the metric deviation from the flat metric $\eta_{\mu\nu}$, which is usually treated as a gauge potential. Unlike the latter, $h_{\mu\nu}(x, c)$ is traceless and satisfies the Hilbert constraint identically, as a manifestation of the absence of unphysically degrees of freedom.

An ansatz for a cubic $L$-$Q$-pair with only first derivatives of $h_{\mu\nu}(x, c)$ yields a unique solution $L_1(c)$ (up to a total derivative). Its obstructions in second order are uniquely cancelled by a quartic interaction $L_2(c)$. Then $L(c) = gL_1(c) + \frac{g^2}{2}L_2(c) + \ldots$ is found to coincide with the beginning of the power series expansion of the Einstein Lagrangian (the Einstein-Hilbert Lagrangian with a quadratic total divergence subtracted), with the classical $h_{\mu\nu}(x)$ replaced by the quantum field $h_{\mu\nu}(x, c)$. Just like the expansion of the Einstein Lagrangian, the series is not expected to terminate with $L_2$ because the power-counting non-renormalizability of quantum gravity cannot be avoided in SQFT.

This finding is considered as the signal (confirmed up to second order) of "diffeomorphism invariant interactions dictated by quantum principles".

What is more: there is another "lock-key" situation, as in QCD. The coupling of matter to helicity 2 through its conserved stress-energy tensor is a unique $L$-$Q$-pair for each type of matter (scalar, matter, Dirac) that admits a local stress-energy tensor. These interactions taken alone exhibit a second-order obstruction that cannot be cancelled. But when the self-interaction of the helicity-2 field is added, the interference terms in second order together with a second-order matter interaction exactly cancel the obstruction.

This is remarkable, for several reasons. First, the same self-interaction cancels the obstructions of each type of matter, separately or in combination. Second, each of the second-order matter interactions (the spin $\frac{1}{2}$ case waiting for confirmation) coincides with the expansion of the classical generally covariant free matter Lagrangians. On the other hand, the obstructions, "calling" for a cancellation by the helicity 2 self-interaction, are due to the violation of Ward identities for the stress-energy tensors. These are intrinsic properties of the matter field in Minkowski spacetime, and do not "know" about its coupling to helicity 2. So, there is a conspicuous "pre-disposition" for generally covariant gravity couplings already present in the flat spacetime QFT of matter fields.

These various findings are assembled under the name "quantum general covariance".

SQFT may be of interest for matter couplings also in another way. It is know from [Weinberg and Witten (1980)] that matter fields of spin or helicity $> 1$ do not admit local conserved stress-energy tensors on their Wigner Hilbert space. This is usually taken as an argument that higher-spin matter cannot couple gravitationally. Yet, it is possible to construct string-localized stress-energy tensors for matter of any spin and helicity [Mund et al. (2017)] (the half-integer case is not worked out). If these can be used in an $L$-$Q$-pair coupling to the equally string-localized gravity potential $h_{\mu\nu}(c)$, one would have a new grip on gravitational couplings of higher-spin matter, without artifices like infinite hierarchies of all spins on Krein spaces. One may again speculate about "Dark Matter".

# 5 SQFT versus BRST

It was mentioned that the BRST method in causal perturbation theory yields very similar constraints on the structure of interactions. There seems to be a deeper reason.

Namely, the BRST variation of all cubic Lagrangians of the Standard Model is a derivative:

$$\delta_{\mathrm{BRST}}(L_1^{\mathrm{BRST}}) = \partial_\mu T_1^\mu, \tag{29}$$

where both $L_1^{\mathrm{BRST}}$ and $T_1^\mu$ are cubic Wick polynomials (including gauge potentials and ghost fields) on their indefinite state space. These derivative terms cause obstructions against BRST invariance of the S-matrix, in the same way as $L$-$Q$-pairs in SQFT cause obstructions against string-independence of the S-matrix. They can also be cancelled in a similar way by higher-order interactions [Scharf (2001)].

One may now add a string-localized derivative term to $L_1^{\mathrm{BRST}}$ to the effect that the resulting string-localized cubic interaction $L_1(c)$ is manifestly BRST-invariant. In particular, the ghost terms are removed. Then, the resulting S-matrix is readily defined on the BRST quotient Hilbert space [Kugo and Ojima (1979)], and the BRST method is redundant. The price is the possible string-dependence. But because $L_1(c) - L_1^{\mathrm{BRST}}$ is a derivative, the two interactions form an $L$-$V$-pair, from which one can deduce the equivalence of the S-matrices, as in Sect. 3.

The difference lies, again, in the conceptual setting: in SQFT one works on the physical Hilbert space with physical degrees of freedom from the start. The second distinction is the fact that SQFT allows to define interacting quantum fields on the same Hilbert space (some of which are string-localized), where BRST would construct a theory without charged fields.

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