How time cures infrared problems

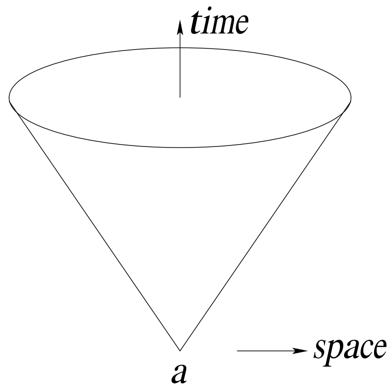
by DETLEV BUCHHOLZ (Göttingen)

Detlev Buchholz received his PhD from the University of Hamburg in 1973. Having held postdoc positions at CERN in Geneva and at the University of California in Berkeley he became Professor of Theoretical Physics at the University of Hamburg in 1979. In 1997 he accepted a call from the University of Göttingen where he stayed until his retirement in 2009. He has established important conceptual and methodical results in quantum field theory such as a collision theory for massless particles, a quantitative description of phase space properties which allowed him to prove the causal independence of observables and existence of thermal equilibrium states in these theories, a comprehensive classification of the possible sector structure and statistics of massive theories and a general method for short distance analysis. From 2000 to 2005 he was Editor–in–Chief of Reviews in Mathematical Physics and he is presently associate editor of three other journals. For his scientific work he was honored by several awards, notably by the Max Planck Medal of the German Physical Society.

The understanding of the sector structure of the physical state space in quantum field theories with long range forces, such as quantum electrodynamics, is a longstanding problem. Recall that a superselection sector is a subspace of the physical Hilbert space of all states of finite energy on which the local observables act irreducibly. So the global superselection observables have sharp values in a sector and the superposition principle holds unrestrictedly there; states in different sectors cannot coherently be superimposed, however. The presence of long range forces leads to an abundance of sectors which carry the same total charge, but differ by multifarious clouds of low energy massless particles which are formed in collisions of the charged particles. In computations one frequently copes with this problem by some *ad hoc* selection of sectors, *e.g.* by picking a convenient physical gauge, and by summing over undetected low energy massless particles. This method provides physically meaningful results but it is conceptually unsatisfactory, for it breaks Lorentz invariance and does not allow the effects of local operations on states to be studied since these do not respect the splitting of the massless particle content into a soft and a hard part because of the uncertainty principle. Celebrated results of quantum field theory such as the PCT theorem, the spin and statistics theorem, collision theory and the determination of the global gauge group from the sector structure, cf. [1], therefore do not apply to these theories.

These conceptual difficulties, which are frequently subsumed under the heading "infrared problems", have received considerable attention in the past, cf. the respective sections in the monographies [1, 2, 3] and references quoted there. In spite of progress on some of its aspects a fully satisfactory solution has not been accomplished to date and, in fact, may never be accomplished along those lines. The infrared problems originate from the unrealistic idea that theory ought to describe experiments in arbitrary regions of Minkowski space which in principle would allow the sectors of infrared clouds to be discriminated. Yet, as a matter of principle, there are no such experiments, so one should not worry about them. Indeed, taking the spacetime limitations of realistic experiments into account, a fully consistent and comprehensive description of the properties of physical states in theories with long range forces was recently established in [4] within the algebraic framework of quantum field theory [1]. We outline here the basic ideas and main results and refer to [4] for precise statements, proofs and further references.

The recent resolution of the infrared problems is based on the insight that the arrow of time should already enter in the interpretation of the microscopic theory. To avoid misunderstandings: the arrow of time is not explained, it is taken into account in the theory as a fundamental empirical fact. Realistic experiments are performed in finite spacetime regions. Beginning at some spacetime point a one performs preparations of states and measurements until sufficient data are taken. In principle, subsequent generations of experimentalists could continue the experiment into the distant future. Thus the maximal regions where data can be taken are future directed lightcones V with apex a whose boundaries are formed by lightrays emanating from a. On the other hand it is

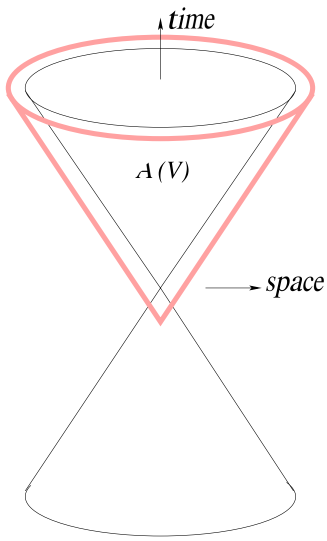


Experiments take place in future directed lightcones V

impossible to make up for missed measurements and operations in the past of the initial point a. The choice of this point is completely arbitrary, one could take for example the birthday and birthplace of Aristotle who invented the term physics ($\phi\upsilon\sigma\iota\kappa\eta$); we all know about this event and reside in the corresponding cone. Or one could take the time and place where the funding of an experiment was approved. What matters is that data in the past of a need not or cannot be taken into account. Phrased differently, it suffices for the comparison of theory and experiment to consider the restrictions of global states to the observables which are localized in a given lightcone V.

We are using here the Heisenberg picture, where the spacetime localization is encoded in the observables. The algebra generated by the observables localized in a given spacetime region $\mathcal{R}$ is denoted by $\mathcal{A}(\mathcal{R})$ and the C*–algebra of all local observables by $\mathcal{A}$. The global states of interest are described by positive, linear and normalized expectation functionals $\omega : \mathcal{A} \rightarrow \mathbb{C}$. Their restrictions $\omega \upharpoonright \mathcal{A}(V)$ to the subalgebras generated by the observables in a given lightcone V contain only partial information about the ensembles and are therefore called partial states.

In theories of exclusively massive particles the algebras $\mathcal{A}(V)$ are known to be irreducible in each sector, hence complete information about the global states can be recovered from the partial states by means of the theory. The situation is markedly different, however, in the presence of massless particles. This is so because outgoing massless particles (radiation) created in the past of a produce no observational effects in V in accordance with Huygens' principle. As a consequence, infrared clouds cannot sharply

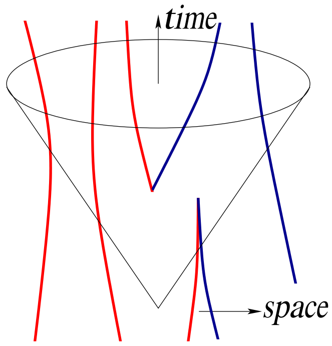


Outgoing massless particles created in the past escape observations in V

be discriminated by measurements in a given lightcone V (all partial states carrying the same total charge are normal with respect to each other). Moreover, the algebras $\mathcal{A}(V)$ are highly reducible. As a matter of fact, their weak closures $\mathcal{A}(V)^-$ in the vacuum sector are factors of type III_1 according to the classification of Connes.

Whereas the infrared clouds appearing in the states cannot sharply be discriminated in any lightcone V, their total charge can be determined there. This follows from the fact that charges are tied to massive particles which eventually enter V, unless they are annihilated or created in pairs carrying opposite charges. These considerations suggest abandoning the concept of superselection sectors of states and replacing it by the coarser notion of charge classes, which combines an abundance of different sectors. Making use of the topologically transitive action of inner automorphisms on the normal states of type III_1 factors, established by Connes and Størmer, this idea has been formalized as follows.

Charge classes: Let ω_1, ω_2 be pure states on the global algebra of observables $\mathcal{A}$. The states belong to the same charge class if, for given lightcone V, there exists some unitary

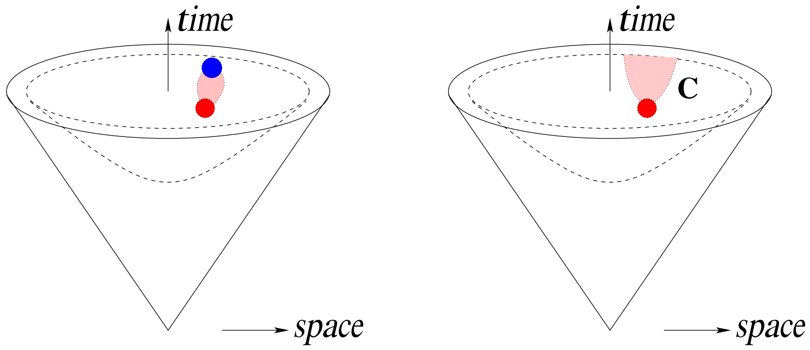


The total charge carried by massive particles can be determined in any V

operator $W_{12} \in \mathcal{A}(V)$ such that $\omega_2 \upharpoonright \mathcal{A}(V) = \omega_1 \circ \mathrm{Ad}\, W_{12} \upharpoonright \mathcal{A}(V)$ or, more generally, if the norm distance between these partial states can be made arbitrarily small for suitable unitaries W_{12}.

Thus the partial states within a charge class can be transformed into each other by physical operations which are described by the adjoint action of unitary operators localized in the lightcones. It can be shown that the definition of charge classes does not depend on the choice of lightcone V; moreover, the corresponding partial states are factorial (primary), *i.e.* all charges which can be determined in V have sharp values within a charge class.

In order to determine the structure of the charge classes of interest one has to understand their mutual relation. In this context it is meaningful to view the lightcones V as globally hyperbolic spacetimes which are foliated by hyperboloids (time shells) playing the role of Cauchy surfaces; thus within V spacelike infinity is formed by its asymptotic lightlike boundary. Given V, one can proceed from the partial states in the charge class of the vacuum state ω_0, carrying zero global charge, to partial states in any given charge class by limits of local operations in V. These operations may be thought of as creation of pairs of opposite charges on some given time shell and the removal of the



Charge creation by creating pairs and removing the unwanted charge within a hyperbolic cone C

unwanted charge on this shell to spacelike infinity; it thereby disappears in the spacelike complement of any relatively compact region in V and thus cannot be observed anymore, leaving behind a charged partial state in V. In order to control the energy required for these operations one has to localize them in broadening hyperbolic cones. A hyperbolic cone C is a convex cone on a time shell in the sense of hyperbolic geometry; its causal completion in V is denoted by $\mathcal{C}$ and called hypercone. These considerations can be put into a mathematically precise form as follows.

Charge creation: Given a charge class, there exists for any hypercone $\mathcal{C} \subset V$ a sequence of inner automorphisms $\{\sigma_n = \mathrm{Ad}\, V_n\}_{n\in\mathbb{N}}$ which are induced by unitary operators $V_n \in \mathcal{A}(\mathcal{C})$ such that the strong limit $\sigma_{\mathcal{C}} \doteq \lim_n \sigma_n$ exists pointwise on $\mathcal{A}(V)$ in the vacuum sector and the partial states $\omega_0 \circ \sigma_{\mathcal{C}} \restriction \mathcal{A}(V)$ obtained by composing the vacuum state with the limit maps belong to the given charge class.

The resulting limit maps $\sigma_{\mathcal{C}} : \mathcal{A}(V) \to \mathcal{A}(V)^-$ are linear, symmetric and multiplicative, *i.e.* they are homomorphisms mapping the algebra $\mathcal{A}(V)$ into its weak closure $\mathcal{A}(V)^-$ in the vacuum sector $\mathcal{H}$. Instead of enlarging this (separable) Hilbert space so as to include charged states it is more convenient to fix $\mathcal{H}$ and to regard the homomorphisms as charge-carrying representations of the observable algebra $\mathcal{A}(V)$ on this space. Their basic properties are summarized in the following proposition.

Proposition 1: Given a charge class, let $\mathcal{C} \subset V$ be any hypercone and let $\sigma_{\mathcal{C}} : \mathcal{A}(V) \to \mathcal{A}(V)^-$ be a corresponding homomorphism defined as above. Then

(a) $\sigma_{\mathcal{C}} \restriction \mathcal{A}(\mathcal{R}) = \iota$ (the identity map) if the regions $\mathcal{R} \subset V$ and $\mathcal{C}$ are spacelike separated.

(b) $\sigma_{\mathcal{C}}\,(\mathcal{A}(\mathcal{R}))^- \subseteq \mathcal{A}(\mathcal{R})^-$ if $\mathcal{C} \subseteq \mathcal{R}$.

(c) The homomorphisms (representations of $\mathcal{A}(V)$) $\sigma_{\mathcal{C}_1}$, $\sigma_{\mathcal{C}_2}$ attached to any given pair of hypercones $\mathcal{C}_1, \mathcal{C}_2 \subset V$ are unitarily equivalent.

Points (a) and (b) encode the information that the homomorphisms arise from local operations in the hypercone $\mathcal{C}$ and are a consequence of the locality of observables (Einstein causality). Point (c) expresses the fact that the sectors of the infrared clouds, which are inevitably produced by charge creating operations, cannot be discriminated by observations in V. In analogy to the terminology used in sector analysis, the maps $\sigma_{\mathcal{C}}$ are called *(hypercone localized) morphisms*. We restrict attention here to the simplest, physically important family of charge classes where in part (b) of the proposition one has equality of the respective algebras. Then one obtains for the weak closures of the observables in the charged representations the equality $\sigma_{\mathcal{C}}(\mathcal{A}(V))^- = \mathcal{A}(V)^-$, so these algebras are again factors of type III_1. In view of the transitivity theorem of Connes and Størmer mentioned above it is therefore meaningful to assume that the morphisms in point (c) of the preceding proposition are related by unitary intertwiners in $\mathcal{A}(V)^-$. We summarize these features for later reference.

Simple charge classes: A charge class is said to be simple if for each hypercone $\mathcal{C} \subset V$ there exist corresponding (simple) localized morphisms $\sigma_{\mathcal{C}}$ such that

(i) $\sigma_{\mathcal{C}}\,(\mathcal{A}(\mathcal{R}))^- = \mathcal{A}(\mathcal{R})^-$ if $\mathcal{R} \supseteq \mathcal{L}$

(ii) for any pair $\sigma_{\mathcal{C}_1}$, $\sigma_{\mathcal{C}_2}$ there exist corresponding unitary operators (intertwiners) $W_{21} \in \mathcal{A}(V)^-$ such that $\sigma_{\mathcal{C}_2} = \mathrm{Ad}\, W_{21} \circ \sigma_{\mathcal{C}_1}$. We write in this case $\sigma_{\mathcal{C}_1} \simeq \sigma_{\mathcal{C}_2}$.

The analysis presented in [4] covers the case of these simple charge classes and thereby the physically most important example of the electric charge. It provides complete results with regard to the problem of charge conjugation, statistics, covariance and the spectral properties of these classes in analogy to sector analysis in massive theories [1]. Similarly to sector analysis, one has to rely on a maximality condition for the hypercone algebras, called hypercone duality:

$$\mathcal{A}(\mathcal{C})' \cap \mathcal{A}(V)^- = \mathcal{A}(\mathcal{C}^c)^- \,, \quad \mathcal{A}(\mathcal{C}^c)' \cap \mathcal{A}(V)^- = \mathcal{A}(\mathcal{C})^- \,,$$

where $\mathcal{C}^c$ denotes the spacelike complement of $\mathcal{C}$ in V and a prime $\,'\,$ at an algebra denotes its commutant in $\mathcal{B}(\mathcal{H})$. Roughly speaking, this condition says that the hypercone algebras cannot be extended without coming into conflict with Einstein causality.

It is an important consequence of hypercone duality that equivalent morphisms which are localized in neighboring hypercones $\mathcal{C}_1, \mathcal{C}_2$ have unitary intertwiners which are contained in $\mathcal{A}(\mathcal{C})^-$, where $\mathcal{C}$ is any larger hypercone containing $\mathcal{C}_1$ and $\mathcal{C}_2$. Making use of this fact and locality one can extend the morphisms from their domain $\mathcal{A}(V)$ to larger algebras (as morphisms). Based on these extensions, the following result describing the structure of simple localized morphisms has been established in [4].

Proposition 2: Let $\sigma_{\mathcal{C}_1}$, $\sigma_{\mathcal{C}_2}$ be simple morphisms which are localized in hypercones $\mathcal{C}_1$ and $\mathcal{C}_2$.

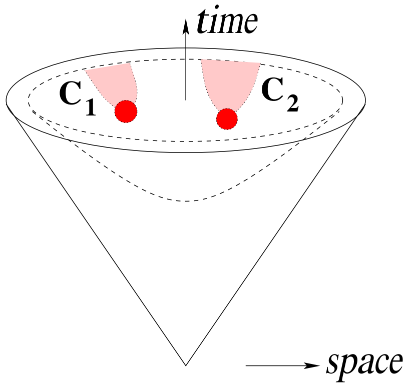
(a) The (suitably extended) morphisms can be composed and there is for any given hypercone $\mathcal{C}$ some simple morphism $\sigma_{\mathcal{C}}$ localized in $\mathcal{C}$ such that $\sigma_{\mathcal{C}_1} \circ \sigma_{\mathcal{C}_2} \simeq \sigma_{\mathcal{C}}$.

(b) $\sigma_{\mathcal{C}_1} \circ \sigma_{\mathcal{C}_2} \simeq \sigma_{\mathcal{C}_2} \circ \sigma_{\mathcal{C}_1}$. If $\sigma_{\mathcal{C}_1}$, $\sigma_{\mathcal{C}_2}$ belong to the same charge class there exists a corresponding intertwiner $\varepsilon(\sigma_{\mathcal{C}_1}, \sigma_{\mathcal{C}_2}) \in \mathcal{A}(V)^-$ which depends only on the given morphisms.

(c) For each charge class there exists a statistics parameter $\varepsilon \in \{\pm 1\}$ such that for any given pair of morphisms $\sigma_{\mathcal{C}_1}$, $\sigma_{\mathcal{C}_2}$ in this class which are localized in spacelike separated hypercones $\mathcal{C}_1, \mathcal{C}_2$ one has $\varepsilon(\sigma_{\mathcal{C}_1}, \sigma_{\mathcal{C}_2}) = \varepsilon\, 1$.

(d) For each simple charge class there exists a simple conjugate charge class such that for any morphism $\sigma_{\mathcal{C}}$ in the given class there is a corresponding morphism $\overline{\sigma}_{\mathcal{C}}$ in the conjugate class satisfying $\sigma_{\mathcal{C}} \circ \overline{\sigma}_{\mathcal{C}} = \overline{\sigma}_{\mathcal{C}} \circ \sigma_{\mathcal{C}} = \iota$. Moreover, the conjugate class has the same statistics parameter as the given class.

According to item (a) the simple charge classes can be composed (the charges can be added) and the composite classes are again simple. Item (b) says that the order of factors in the composition does not matter, one always ends up in the same class. According to (c) the states in each charge class have definite (Bose, respectively Fermi) statistics. It is encoded in the statistics parameter which is the value of the group theoretic commutator



Spacelike separated hyperbolic cones

of cone–localized charged field operators resulting from the morphisms. Item (d) says that for each simple charge class there is a simple conjugate class of states carrying opposite (neutralizing) charges with the same statistics. Finally, items (a), (b) and (d) imply that the equivalence classes of simple morphisms determine an abelian group with product given by composition and unit element ι. Its dual is the global (abelian) gauge group generated by the simple charges. Since these results do not depend on the choice of V, these physically important data can be determined in any lightcone, in contrast to the superselection sectors of the infrared clouds. Thus the elusive theoretical effects of these clouds completely disappear from the discussion by taking into proper account the limitations on real experiments imposed by the arrow of time.

In the analysis of the covariance and spectral properties of the simple charge classes one is faced with the problem that the spacetime symmetry group, the Poincaré group $\mathcal{P}_+^\uparrow$, does not leave any lightcone invariant. One therefore considers for given V its sub–semigroup $\mathcal{S}_+^\uparrow = \overline{V}_+ \rtimes \mathcal{L}_+^\uparrow$, where $\mathcal{L}_+^\uparrow$ is the group of proper orthochronous Lorentz transformations leaving the apex of V fixed and $\overline{V}_+$ the semigroup of future directed timelike and lightlike translations. This semigroup acts by endomorphisms on V and induces corresponding endomorphisms $\alpha_{\,\cdot}$ of $\mathcal{A}(V)^-$ which transform the local observables in accordance with the underlying geometric action, *i.e.* $\alpha_\lambda(\mathcal{A}(\mathcal{R})^-) = \mathcal{A}(\lambda\mathcal{R})^-$ for all regions $\mathcal{R} \subset V$ and $\lambda \in \mathcal{S}_+^\uparrow$. The following characterization of covariant morphisms is appropriate in this situation.

Covariant simple morphisms: Let $\sigma : \mathfrak{A}(V) \to \mathfrak{A}(V)^-$ be a simple morphism. (In order to simplify notation its localization hypercone will be omitted in the following.) The morphism is said to be covariant if there exists a family of equivalent morphisms $\{{}^\lambda\sigma : \mathcal{A}(V) \to \mathcal{A}(V)^-\}_{\lambda \in \mathcal{S}_+^\uparrow}$ with ${}^1\sigma = \sigma$ and a strongly continuous family of unitary operators $\lambda \mapsto \Gamma_\lambda \in \mathcal{A}(V)^-$ such that $\alpha_\lambda(\Gamma_\mu)$ are intertwiners between ${}^{\lambda\mu}\sigma$ and ${}^\lambda\sigma$ for $\lambda, \mu \in \mathcal{S}_+^\uparrow$.

The morphisms ${}^\lambda\sigma$ describe a situation where the charge created by σ is in addition shifted by λ. The idea that this transport of charges can be accomplished in a covariant manner enters in the condition that the unitaries Γ_μ inducing the transport from σ to ${}^\mu\sigma$ are mapped by the action of the semigroup to operators $\alpha_\lambda(\Gamma_\mu)$ which induce the transport of the shifted charges, *i.e.* from ${}^{\lambda}\sigma$ to ${}^{\lambda\mu}\sigma$. The properties of the covariant morphisms are described in the following proposition established in [4].

Proposition 3: Let the lightcone V be given and consider the subfamily of all simple hypercone localized morphisms $\sigma : \mathcal{A}(V) \to \mathcal{A}(V)^-$ which are also covariant.

(a) The subfamily is stable under composition and conjugation.

(b) Each morphism σ determines a unique continuous unitary representation U_σ of (the covering of) the full Poincaré group $\widetilde{\mathcal{P}}_+^\uparrow = \mathbb{R}^4 \rtimes \widetilde{\mathcal{L}}_+^\uparrow$ such that

$$\mathrm{Ad}\, U_\sigma(\tilde{\lambda}) \circ \sigma = \sigma \circ \alpha_\lambda \,, \quad \widetilde{\lambda} \in \overline{V}_+ \rtimes \widetilde{\mathcal{L}}_+^\uparrow \,,$$

where $\widetilde{\lambda} \mapsto \lambda$ is the canonical covering map from the covering group to the Poincaré group.

(c) $\mathrm{sp}\, U_\sigma \upharpoonright \mathbb{R}^4 \subset \overline{V}_+$, *i.e.* the joint spectrum of the generators of spacetime translations satisfies the relativistic spectrum condition.

This result shows that the covariant morphisms describe the physically expected properties of elementary systems in a meaningful manner. There is for each given charge class and lightcone V an (up to equivalence) unique continuous unitary representation of $\widetilde{\mathcal{P}}_+^\uparrow$. Thus, restricting observations to lightcones one does not encounter the spontaneous breakdown of the Lorentz group, met in Minkowski space, and can interpret the generators of the representation as energy, angular momentum, *etc.*, of the underlying partial states. The energy is bounded from below, expressing the stability of the charged states. We mention as an aside that this feature is inherited from the vacuum sector where this property holds by definition since the vacuum is a ground state for all inertial observers. It has to be noticed, however, that the generators of the time translations should not be interpreted as genuine observables in the presence of massless particles since in that case they are not affiliated with the algebra of observables $\mathcal{A}(V)^-$. This can be understood if one bears in mind that part of the energy content of the global states will be lost by outgoing radiation created in the past of V. Phrased differently, the energy content of the partial states on $\mathcal{A}(V)$ is fluctuating, and the generators of the time translations subsume this effect in a gross manner, akin to the generators (Liouvillians) in quantum statistical mechanics.

So, to summarize, the notorious infrared problems in the interpretation of theories with long range forces originate from the unreasonable idealization of observations covering all of Minkowski space. Observations and operations are at best performed in future directed lightcones, hence the arrow of time enters already in the interpretation of the microscopic theory. The restriction of global states to the observables in a given lightcone V amounts to a geometric infrared regularization. Instead of splitting the massless particle content into an energetically soft and a hard part, it is split into a marginal part which escapes observations in V and an essential part which can be observed and manipulated in V but which does not allow discrimination of infrared sectors. This splitting is compatible with the Lorentz symmetry and Einstein Causality and therefore allows the statistics, charge conjugation, covariance and spectral properties of the charge classes to be determined. The general method established in [4] covers so far only simple charges related to an abelian gauge group, such as the electric charge. But work in progress indicates that it may be improved so as to apply to all charge classes.

References

[1] R. Haag, *Local Quantum Physics: Fields, Particles, Algebras*, Berlin, Springer (1992)

[2] O. Steinmann, *Perturbative Quantum Electrodynamics and Axiomatic Field Theory*, Berlin, Springer (2000)

[3] F. Strocchi, *An Introduction to Non–Perturbative Foundations of Quantum Field Theory*, Oxford University Press (2013)

[4] D. Buchholz and J.E. Roberts, New light on infrared problems; Sectors, statistics, symmetries and spectrum. e-Print arXiv:1304.2794 (to appear in Commun. Math. Phys.)